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Abstract

We propose a wavelet-based approach for construction of a financial cycle proxy. Specifically, we decompose three key macro-financial variables – private credit, house prices, and stock prices – on a frequency-scale basis using wavelet multiresolution analysis. The resulting "wavelet-based" sub-series are aggregated into a composite index representing our cycle proxy. Selection of the sub-series deemed most relevant is done by emphasizing early warning properties. The wavelet-based financial cycle proxy is shown to perform well in detecting banking crises in out-of-sample exercises, outperforming the credit-to-GDP gap and a financial cycle proxy derived using the approach of Schüler et al. (2015).

Keywords: financial cycle, early warning indicators, wavelets, multiresolution analysis.
JEL classification: C49, E32, E44.

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1 Introduction

As links between financial markets and real economic activity has attracted much interest during the years after the 2007-2008 global financial crisis, characterization of the so-called financial cycle has been re-discovered as a topic of research. In contrast to its well-known cousin the business cycle, financial cycle still remains a phenomenon without a generally accepted characterization. One definition for financial cycle comes from Borio (2014), according to whom financial cycles can be thought to reflect "self-reinforcing interactions between perceptions of value and risk, attitudes towards risk and financing constraints, which translate into booms followed by busts." This definition encompasses the analytical properties of a financial cycle, but is as such hard to grasp in empirical terms. In practice, we can understand financial cycle as medium-term co-movement of credit and property prices (Drehmann et al., 2012). Indeed, relying on this definition authors argue that the financial cycle is a distinct phenomenon from the business cycle. Further, Borio (2014) finds peaks in financial cycles to be closely associated with financial/banking crises.[1] Thus, knowing the current phase of the financial cycle is important for reasons such as a) better understanding of the underlying economic activity as well as b) detection of accumulating imbalances that possibly lead to costly financial crises.

In this paper we propose a wavelet approach for constructing a proxy for the financial cycle for 13 European countries. First we decompose three relevant macro-financial variables – private credit, house prices, and stock prices – on frequency-scale basis using wavelet multiresolution analysis. Obtained sub-series represent movements at different period lengths, or equivalently at different frequencies. In similar fashion to Gallegati (2013), we can aggregate a selection of the sub-series into a parsimonious composite index, which will constitute our financial cycle proxy. Sub-series deemed most relevant for the composite index is done by emphasizing the resulting early warning properties. Specifically, we run full sample pooled logit early warning regressions using Babecký et al. (2012) banking crisis periods as explained variable in order to characterize which wavelet scales are most helpful in detecting financial crises. The results show that we can identify a robust financial cycle with most important cyclicality taking place at ranges of 4-8 and 8-16 years. We show that compared to two similar measures, namely the credit-to-GDP gap as well as another financial cycle proxy constructed using the approach introduced by Schüler et al. (2015), our wavelet-based proxy outperforms in out-of-sample detection of financial crises. These results suggest that the wavelet-based financial cycle proxy makes an useful addition to the macroprudential toolkit of central banks.

Rest of the paper is organized as follows. In section 2 we describe data collection and treatment. Section 3 explains construction of the financial cycle proxy. Section 4 presents results from two out-of-sample exercises. In section 5 we perform robustness checks using an alternative crisis dataset. Finally, section 6 concludes.

[1] In this text we use terms financial crisis and banking crisis synonymously.

2 Data

The desired property of our financial cycle proxy is to predict financial crises. Hence, we want to include variables whose swings have the most considerable effect to the macro-financial sector as a whole. This is done by selecting three key variables based on their frequency of appearance in the junction of literature dealing with financial cycles and early warning indicators.[2]

Similarly to Drehmann et al. (2012) we include private credit and house prices as our first two variables. Credit is a common guest in literature revolving around financial stability. Works such as Aikman et al. (2015), Schularick and Taylor (2012), and Geanakolpos (2010) have stressed the role of credit in determining swings in the financial conditions and asset prices. Importance of credit has not gone unnoticed in policy work either as it has been firmly baked into the Basel III regulation in the form of credit-to-GDP gap. On the other hand, the burst of the U.S. real-estate bubble in 2007-2008 exhibited how dire an effect negative developments in the residential sector can have on the economy. Third variable we include is local benchmark stock index as swings in stock prices are typically associated with boom-bust cycles in the financial markets. Although Drehmann et al. (2012) find stock prices to be less-fitting when talking about financial cycles, later research by Schüler et al. (2015) has challenged this view by claiming that stock prices do share important common cyclicality with credit and residential prices. Further, stock returns are often included in early warning exercises (Tölö et al., 2017).

We include 13 countries in our analysis: Austria, Belgium, Denmark, Germany, Finland, France, Ireland, Italy, Netherlands, Portugal, Spain, Sweden, and United Kingdom.[3] Series are reported at quarterly frequency. The complete time span ranges from 1970Q1 to 2016Q2, but there are differences across countries in data availability: Eight countries have data from the start of 1970's, otherwise starting dates are in the 1980's. Tables 10 - 12 in Appendix provide information on the raw time series and Table 1 showcases the common time range between variables for each country. As usual in related literature (e.g. Gallegati, 2008 and Rua, 2011), series are deflated using country-specific CPI index[4] and log-differenced to obtain comparable units across variables.

[2] A summary of the most common variables in early warning exercises can be found from Tölö et al. (2017).
[3] AUT, BEL, DNK, DEU, FIN, FRA, IRL, ITA, NLD, PRT, ESP, SWE, and GBR.
[4] Credit and stock prices are deflated, whereas residential prices are originally reported in real terms.

Country	Common time sample
AUT	1986Q4-2016Q2
BEL	1985Q3-2016Q2
DEU	1970Q2-2016Q2
DNK	1983Q2-2016Q2
ESP	1985Q2-2016Q2
FIN	1971Q1-2016Q2
FRA	1970Q2-2016Q2
IRL	1971Q3-2016Q1
ITA	1970Q4-2016Q2
NLD	1970Q2-2016Q2
PRT	1988Q2-2016Q2
SWE	1970Q2-2016Q2
GBR	1970Q2-2016Q2

Table 1: Common time interval of private credit, house prices, and stock prices for each sample country (after differencing and possible deflation).

3 Construction of wavelet-based financial cycle proxy

3.1 Wavelet decomposition using multiresolution analysis

Our aggregation of individual variables to a financial cycle proxy is in the spirit of Gallegati (2013) who constructed a market stress index (with daily frequency) using a similar technique. Let us denote our three series as

$$I_1 : \text{ Private credit}$$

$$I_2 : \text{ House prices}$$

$$I_3 : \text{ Stock prices}$$

where each I_i is to be understood as a finite sample from a time series. For each I_i we perform a *maximum overlap discrete wavelet transform* (MODWT) multiresolution decomposition of level $J = 6$.[5] The wavelet filter chosen is the Daubechies' least-asymmetric wavelet with 4 vanishing moments.[6] Further, we use symmetric reflection principle to mitigate end-point problems. The decomposition of I_i at each period t reads[7]

[5]For a detailed discussion about discrete wavelet transform and multiresolution analysis reader is instructed to Percival and Walden (2000). Furthermore, Faria and Verona (2017) present a primer on the subject.

[6]In Matlab's Wavelet Toolbox this wavelet is referred to as "sym4". We experimented with different wavelet filters as well as varying the amount of vanishing moments but all specifications yielded more or less the same results.

[7]We omit the time subscripts for notational convenience.

$$I_i = \sum_{j=1}^{J} D_{j,i} + S_{J,i} \quad , \; J = 6 \tag{1}$$

In equation (1) each $D_{j,i}$ - called *wavelet details* - represent movements in I_i at different period bands (or equivalently frequency bands). With quarterly data, $1st$ wavelet detail represents movements at 2-4 quarters, $2nd$ at 1-2 years, $3rd$ at 2-4 years, $4th$ at 4-8 years, $5th$ at 8-16 years, and $6th$ at 16-32 years. $S_{6,i}$ is called *wavelet smooth* and it represents movements at all period lengths from 32 years onwards. We decide to use $J = 6$ in order to include period lengths up to 32 years as previous literature (Drehmann et al., 2012) has suggested 30 years as the upper bound for relevant financial cycle periodicities. For notational convenience we will not distinguish between wavelet details and the wavelet smooth, but decide to call them commonly as *wavelet scales*. That is, equation (1) is rewritten as

$$I_i = \sum_{j=1}^{7} WS_{j,i} \tag{2}$$

where $WS_{j,i} = D_{j,i}$ for $j = 1, \ldots 6$ and $WS_{7,i} = S_{6,i}$.

After decomposing our series into wavelet scales series $WS_{j,i}$ we aggregate them into a *composite index* (CI) representing our financial cycle proxy. The way we will approach this task is first to assign weights $\omega_{j,i}$ for each $WS_{j,i}$, and then at given scale level j sum $WS_{j,i}$ together into sub-indices CI_j, $j = 1, \ldots, 7$. As our aim is to construct a parsimonious financial cycle proxy, we will only consider weights $\omega_{j,i} = \{0, 1\}$ for all i and j. This means that in each sub-index CI_j we either include wavelet scale $WS_{j,i}$ fully (weight 1) or not at all (weight 0). Formally, the sub-indices are given by

$$CI_j = \sum_i \omega_{j,i} WS_{j,i} \quad \text{for } j = 1, \ldots 7 \tag{3}$$

The final composite index CI is obtained by summing together all sub-indices CI_j, that is

$$CI = \sum_j CI_j \tag{4}$$

There exists many different versions of the composite index CI depending on our choice for the combination of weights $\omega_{j,i}$, and we wish to distinguish different composite indices from each other. To achieve this, we represent an arbitrary combination of weights as matrix Ω:

$$\Omega = \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \\ \omega_{41} & \omega_{42} & \omega_{43} \\ \omega_{51} & \omega_{52} & \omega_{53} \\ \omega_{61} & \omega_{62} & \omega_{63} \\ \omega_{71} & \omega_{72} & \omega_{73} \end{bmatrix} \quad , \ \omega_{i,j} = \{0, 1\} \ \forall \, i, j$$

First column in Ω represents weights assigned to I_1 (credit) at all seven wavelet scale levels. Similarly, second column represents weights assigned to I_2 (house prices) and third column weights assigned to I_3 (stock prices). Ω belongs to the set of all possible weights combinations $\mathcal{W}$, i.e. $\Omega \in \mathcal{W}$. Now, to distinguish for composite index CI that is built by weight set Ω, we simply write $CI(\Omega)$. That is, $CI(\Omega)$ is constructed as in equations (3) and (4) using a given weight set Ω. The crucial part of the analysis is to determine Ω that yields the optimal combination, a question that we turn to next.

3.2 Determining optimal composition for the wavelet-based financial cycle proxy

As the wavelet sub-series $WS_{j,i}$ represent movements of I_i in certain period length band (or equivalently frequency band), choosing weights $\omega_{j,i}$ essentially fixes at what period lengths the resulting cycle proxy will operate. Thus, the task of selecting optimal weight set boils down to the question at which period lengths would we want our financial cycle proxy to operate.

To gain insights on this question we can turn to the earlier literature about financial cycles. Claessens et al. (2011) and Aikman et al. (2015) have found that cycles in credit (and in the former also in house prices) tend to be longer than those of the business cycle. Further, Drehmann et al. (2012) and Borio (2014) have emphasized mid-term (8-30 years) co-movement between credit and house prices as basis for the financial cycle. Focusing solely on the U.S., contribution by Verona (2016) (also using wavelet methods) supports the importance of mid-range period lengths among credit, house prices, and stock prices when scrutinized individually. However, Schüler et al. (2015) emphasize the importance of multivariate treatment and claim that there can exist significant heterogeneity in the lengths of the cycles across countries, especially in the lower end of the period length band. Using the same set of countries as in our analysis, authors find that for some countries there exists relevant cyclicality also below period length of 8 years. Thus, based on earlier literature we can hypothesize that for our analysis especially relevant wavelet scale levels are 4, 5, and 6, representing period lengths of 4-8, 8-16, and 16-32 years, respectively. This view is supported by examining the average wavelet coherencies between the three series I_i , $i = 1, 2, 3$, which reveals that most consistent co-movement between them is located at period lengths corresponding to scales 4 and above. The wavelet coherence analysis is presented in Section A.2 of the Appendix.

In addition to the evidence presented so far, we would like to have a quantifiable objective according to which to select the optimal set of weights. As discussed in Section 1, in this paper we set the main desired property of the financial cycle proxy to be early detection of financial crises. Thus, we can choose the final set of weights on the basis of resulting early warning properties of the financial cycle proxy. This is achieved by running pooled logit-regressions where we use different compositions of CI to explain known periods of banking crises.

In formal terms, we construct crisis dummy series $\{y\}_t$ indicating crisis periods for each country in the sample. To identify crisis periods we use the banking crisis dataset compiled by Babecký et al. (2012).[8] For example, if a country experienced a banking crisis at certain period t, then $y_t = 1$, and if there was no crisis at period t, then $y_t = 0$. Following common practice in early warning literature (e.g. Schularick and Taylor, 2012, Kaminsky and Reinhart, 1999, and Lo Duca and Peltonen, 2013), instead of explaining actual crises periods we will try to explain some time range preceding the crisis. This time range is referred to as the *vulnerability horizon*. That is, we create a transformed crisis dummy series $\{\hat{y}\}_t$ where values 1 indicate that there will be a crisis in the given vulnerability horizon, and values 0 that there will be no crisis in the given vulnerability horizon. We set the baseline vulnerability horizon to be 2-4 years (8-16 quarters). We also test for two other vulnerability horizons, 1-3 years (4-12 quarters) and 3-5 years (12-20 quarters). As an example, if we know that for a given country at period t^* we have $y_{t^*} = 1$ and for every other period $y_t = 1$, then with baseline vulnerability horizon of 8-16 quarters it holds that $\hat{y}_{t^*-16},\ \hat{y}_{t^*-15}, \ldots, \hat{y}_{t^*-8}$, will equal to 1. For any other period $\hat{y}_t$ will equal to zero.

In order to treat biases arising during crisis situations, we remove from the sample periods that actually witnessed a crisis as well as 6 periods following a crisis.[9] That is, if $y_{t^*=1}$, then periods $t^*, t^* + 1, \ldots, t^* + 6$ will be removed from the sample. Furthermore, we also drop observations left between the vulnerability horizon and start of the crisis as these observations might also introduce an unwanted bias. The amount of dropped periods preceding a crisis will depend on the lower end of the vulnerability horizon.

Next, we run a pooled logit-regressions using full available sample, where we use different $CI(\Omega)$ as the sole explanatory variable (plus intercept) to explain $\hat{y}$.[10] For discussion on the appropriateness of a pooled approach, see e.g. Demirgüç-Kunt and Detragiache (2000). The logistics regression with one explanatory variables reads

$$p \equiv P(\hat{y} = 1) = \frac{e^{\beta_0 + X\beta_1}}{1 + e^{\beta_0 + X\beta_1}} \tag{5}$$

[8]The Babecký et al. (2012) crisis dataset, or also commonly referred to as the ESCB Heads of Research crisis dataset, aggregates information about crises periods from several influential papers. Banking crises are defined as periods that exhibit significant signs of financial distress as evidenced by bank runs or losses. For more information see Section A.3 in the Appendix.

[9]See e.g. Behn et al. (2013) and Bussière and Fratzscherm (2006).

[10]Although the cycle proxies are estimated for the full interval 1970Q1-2016Q2, the regression is performed on a slightly shorter interval depending on the higher end of the vulnerability horizon. For example, if we use the baseline vulnerability horizon of 8-16 quarters, the last observation to be included in the regression sample is 2012Q2 as we cannot see into the future (beyond 2016Q2) for looming crises.

where X contains pooled values of our financial cycle proxies. After obtaining estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ we use them to construct model implied in-sample crisis probabilities for each country k at time t according to

$$\hat{p}_{t,k} = \frac{e^{\hat{\beta}_0 + CI_{t,k}(\Omega)\hat{\beta}_1}}{1 + e^{\hat{\beta}_0 + CI_{t,k}(\Omega)\hat{\beta}}} \tag{6}$$

We obtain binary predictions for crises if the model implied probability exceeds some threshold τ, that is

$$P_{t,k}(\tau) = \begin{cases} 1 & \text{if } \hat{p}_{t,k} > \tau \\ 0 & \text{if } \hat{p}_{t,k} \leq \tau \end{cases}$$

Using the binary predictions we can calculate usual early warning signalling statistics with different threshold values τ. First, the *confusion matrix* observations *true positive* (TP), *true negative* (TN), *false positive* (FP), and *false negative* (FN) are given as

	$P_{t,k}(\tau) = 1$	$P_{t,k}(\tau) = 0$
$\hat{y}_{t,k} = 1$	$TP(\tau)$: Correct alarm	$FN(\tau)$: Missed crisis (Type II error)
$\hat{y}_{t,k} = 0$	$FP(\tau)$: False alarm (Type I error)	$TN(\tau)$: Correctly no alarm

In turn, confusion matrix observations allow us to derive *true positive rate* (TPR) and *false positive rate* (FPR), defined as

$$TPR(\tau) = \frac{TP(\tau)}{TP(\tau) + FN(\tau)}$$
$$FPR(\tau) = \frac{FP(\tau)}{FP(\tau) + TN(\tau)}$$

Now imagine we were to plot $TPR(\tau)$ and $FPR(\tau)$ for different values of τ in a FPR-TPR plane. The resulting curve goes by the name *receiver operating characteristic* (ROC) curve, and the area under the ROC curve is referred to as the *area under receiver operating characteristic curve* (AUROC). AUROC tells us how well the model explains banking crises over different values of the threshold τ. The greater the AUROC, the higher the ratio $\frac{TPR}{FPR}$ is on average. AUROC alone might not always be a good metric for evaluating early warning properties as there might be just a few values of τ that produce a high $\frac{TPR}{FPR}$ ratio, in which case AUROC will generally not be high. As an attempt to mend this deficiency in AUROC we introduce a notion of *usefulness to the policymaker*. First, define a *loss function* as

$$L \equiv \theta \frac{FN}{TP + FN} + (1 - \theta) \frac{FP}{FP + TN} \tag{7}$$

where θ is a parameter dictating the policymaker's relative risk aversion between type I and type II errors. When θ equals to 0.50, then the policymaker treats both type I and II errors as equally bad outcomes. If the policymaker is interested in correctly predicting more crises periods with the cost of more false flags (type I errors), then she will set $0.50 < \theta < 1$. On the other hand, if the policymaker is concerned on not giving too many false alarms with the cost of more often missing a crisis (type II errors), then she will set $0 < \theta < 0.50$.

In their seminal work, Alessi and Detken (2011) defined *absolute usefulness* as

$$U_a = min\{\theta, 1-\theta\} - L$$

The higher the U_a, the more useful an indicator is for policy maker as an early warning tool. As suggested by Sarlin (2013), we can present the idea of usefulness as a ratio that compares our model to the perfect model, making it easier to interpret. In a perfect model $L = 0$, implying that absolute usefulness equals to $min(\theta, 1-\theta)$. Thus, *relative usefulness* is defined as[11]

$$U_r = \frac{U_a}{min(\theta\ ,\ 1-\theta)}$$

In Table 2 we present results from the full sample early warning exercise for the following compositions of the wavelet-based cycle proxy:

$$\begin{aligned}
CI(\Omega_{j^*}):&\quad \Omega_{j^*} = \{\omega_{j,i} \mid \omega_{j^*,i} = 1, \text{otherwise } \omega_{j,i} = 0\} \quad \text{where } j^* = 1, 2, ..., 6 \\
CI(\Omega_{45}):&\quad \Omega_{45} = \{\omega_{j,i} \mid \omega_{h,i} = 1 \text{ for } h = 4, 5\ ,\ \text{otherwise } \omega_{j,i} = 0\} \\
CI(\Omega_{56}):&\quad \Omega_{56} = \{\omega_{j,i} \mid \omega_{h,i} = 1 \text{ for } h = 5, 6\ ,\ \text{otherwise } \omega_{j,i} = 0\} \\
CI(\Omega_{456}):&\quad \Omega_{456} = \{\omega_{j,i} \mid \omega_{h,i} = 1 \text{ for } h = 4, 5, 6\ ,\ \text{otherwise } \omega_{j,i} = 0\}
\end{aligned}$$

That is, first we run the early warning exercise by selecting all wavelet scales individually for construction of the composite index. For example, $CI(\Omega_4)$ means that in (3) we assign weights equal to 1 for the scale level 4 of each variable I_i, $i = 1, 2, 3$. For any other wavelet scale/variable combination we assign the weight zero. Second, we use different combinations of the medium-length wavelet scales. For example, in $CI(\Omega_{456})$ we assign weights equal to 1 for wavelet scales 4, 5, and 6 across all three variables, and for any other scale/variable combination weights equal to zero.

[11] Sarlin (2013) also extends the loss function of (Alessi and Detken, 2011) to explicitly take into account unconditional sample crisis probabilities in order to correct for biases arising from the fact that tranquil times are much more common than crises periods. However, Alessi and Detken (2014) defend their less-complicated approach by arguing that introducing relative sample sizes to the loss function, and thus to the usefulness measure, is "not robust to minor changes in preferences or in the probability of crises". We opt not to introduce unconditional sample probabilities.

	$CI(\Omega_1)$	$CI(\Omega_2)$	$CI(\Omega_3)$	$CI(\Omega_4)$	$CI(\Omega_5)$	$CI(\Omega_6)$	$CI(\Omega_7)$	$CI(\Omega_{45})$	$CI(\Omega_{56})$	$CI(\Omega_{46})$	$CI(\Omega_{456})$
					Vulnerability horizon 4-12 quarters						
Intercept	$\mathbf{-1.9}$	$\mathbf{-1.9}$	$\mathbf{-1.9}$	$\mathbf{-2.2}$	$\mathbf{-2.1}$	$\mathbf{-1.9}$	$\mathbf{-1.9}$	$\mathbf{-2.3}$	$\mathbf{-2.1}$	$\mathbf{-2.2}$	$\mathbf{-2.2}$
CI	0.3	0.8	2.15	**24.5**	**29.4**	$\mathbf{-16.3}$	0.0	**18.8**	**14.0**	**17.2**	**14.6**
AUROC	0.50	0.47	0.51	0.64	0.61	0.51	0.50	0.64	0.59	0.63	0.64
U_r	0	0.02	0.04	0.21	0.24	0.09	0	0.23	0.17	0.20	0.22
Obs	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500	1500
					Vulnerability horizon 8-16 quarters						
Intercept	$\mathbf{-1.8}$	$\mathbf{-1.8}$	$\mathbf{-1.9}$	$\mathbf{-2.1}$	**2.1**	$\mathbf{-1.9}$	$\mathbf{-1.8}$	$\mathbf{-2.2}$	$\mathbf{-2.1}$	$\mathbf{-2.1}$	$\mathbf{-2.2}$
CI	0.8	0.7	*-5.0*	**24.4**	**37.9**	3.1	-0.4	**21.1**	**23.4**	**20.0**	**17.9**
AUROC	0.49	0.47	0.55	0.64	0.65	0.52	0.50	0.66	0.62	0.63	0.65
U_r	0.03	0.01	0.09	0.25	0.25	0.05	0	0.27	0.17	0.27	0.25
Obs	1364	1364	1364	1364	1364	1364	1364	1364	1364	1364	1364
					Vulnerability horizon 12-20 quarters						
Intercept	$\mathbf{-1.8}$	$\mathbf{-1.8}$	$\mathbf{-1.8}$	$\mathbf{-1.8}$	$\mathbf{-1.9}$	$\mathbf{-1.9}$	$\mathbf{-1.8}$	$\mathbf{-1.9}$	$\mathbf{-2.0}$	$\mathbf{-1.9}$	$\mathbf{-1.9}$
CI	-0.1	1.8	*5.6*	**7.2**	**23.1**	**21.3**	-0.3	**9.2**	**19.2**	**9.2**	**9.5**
AUROC	0.51	0.51	0.54	0.53	0.61	0.51	0.51	0.58	0.57	0.53	0.56
U_r	0.02	0.02	0.06	0.06	0.18	0.09	0.02	0.13	0.14	0.08	0.13
Obs	1236	1236	1236	1236	1236	1236	1236	1236	1236	1236	1236

Table 2: *Full sample early warning results using different weight selections for the wavelet-based financial cycle proxy CI. Explained variable is a dummy vector where values equal to 1 indicate Babecký et al. (2012) banking crises at different vulnerability horizons. Top block refers to vulnerability horizon of 1-3 years, middle block 2-4 years, and bottom block 3-5 years. Usefulness measure U_r is derived using policymaker's preference parameter value $\theta = 0.50$. Wavelet estimations are done using full sample 1970Q1 - 2016Q2. For some countries series are shorter due to data availability issues. The last period for the pooled logistic regression will be either 2013Q2, 2012Q2, or 2011Q2, depending on the vulnerability horizon. Statistical significance of intercept and cycle coefficients at 5% and 1% levels are indicated by italic letters and bold letters, respectively.*

Based on the results for the full sample early warning regression shown in Table 2 we draw several conclusions. First, scales 1 and 2 are useless as the ability of $CI(\Omega_1)$ and $CI(\Omega_2)$ to detect banking crises is virtually a coin-toss at each vulnerability horizon. Second, scales 4 and 5 are the most relevant individual scales for our financial cycle proxy. From all $CI(\Omega_i)$, $i = 1, \ldots, 7$ only $CI(\Omega_4)$ and $CI(\Omega_5)$ have statistically significant coefficients at all three vulnerability horizons. They also yield by far the best results in terms of AUROC and relative usefulness.[12] Scale 3 is statistically significant (5% confidence level) at 8-16 and 12-20 quarter vulnerability horizons, but in the former the sign is wrong (negative).[13] In the latter horizon the sign is positive but AUROC and relative usefulness remain low. Coefficient for scale 6 is strongly negative in the 4-12 quarter vulnerability horizon, and remains statistically insignificant at 8-16 quarter vulnerability horizon. At the more distant 12-20 quarter vulnerability horizon $CI(\Omega_6)$ has a statistically significant and positive coefficient, but again AUROC and relative usefulness remain low. Scale 7 is too smooth to be of any practical importance for our early warning setting, which can be seen from insignificant coefficients as well as AUROC and relative usefulness pointing to a coin-toss situation.

[12] With the exception that at the more distant vulnerability horizon of 12-20 quarters changes in $CI(\Omega_4)$ are already too erratic to predict crises, resulting in low AUROC and relative usefulness (0.53 and 0.06, respectively).

[13] We require that an increase in financial cycle translates to increased risk of financial crises.

Results for different combinations of wavelet scales 4, 5, and 6 are presented in the last four columns of Table 2. We see that $CI(\Omega_{45})$ yields the best results in vulnerability horizons of 4-12 and 8-16 quarters in terms of AUROC and relative usefulness. Results for cycle $CI(\Omega_{456})$ are almost as good as for $CI(\Omega_{45})$, but we see that adding scale 6 to the cycle proxy does little good; rather, it merely diminishes the success of CI as an early warning indicator. In 12-20 quarter vulnerability horizon $CI(\Omega_{45})$ again yields highest AUROC, but $CI(\Omega_{56})$ produces a slightly higher relative usefulness. The difference is however tiny (0.13 vs. 0.14), and the AUROC as well as relative usefulness are considerably lower than the overall highest AUROC and relative usefulness (0.66 and 0.27, respectively) obtained with $CI(\Omega_{45})$ at the 8-16 quarter vulnerability horizon. In summary, we deduce that scales 4 and 5 are the only particularly relevant wavelet scales as far as early warning properties of the resulting wavelet-based financial cycle proxy are concerned, which leads us to choose $CI(\Omega_{45})$ as our final cycle proxy.

Figure 1 offers a visual inspection of $CI(\Omega_{45})$ for all sample countries, with Babecký et al. (2012) crisis periods plotted as yellow shaded areas. We present two versions of the cycle: one estimated with full sample and another derived on an expanding sample. "Expanding sample" means that cycles are estimated recursively adding one new observation to the wavelet estimation interval, and then saving the last observations of the estimated cycles. The series of last estimated observations constitutes the expanding sample series for a given country. We use 10 years worth of data in the first estimation of the expanding sample wavelet cycles.[14]

[14]Matlab's Wavelet Toolbox determines the maximum MODWT multiresolution decomposition level for given input length L using the rule $\lfloor log_2(L) \rfloor$. Choosing 10 years of data in the pre-sample means 40 quarterly observations. This enables us to perform level 5 decomposition as $\lfloor log_2(40) \rfloor = 5$, which is enough to estimate five wavelet details (scales 1-5), with wavelet smooth (scale 6) representing period length band from 16 years to infinity. Note that although in Section 3.1 we used used level 6 MODWT decomposition, the results here are equivalent since we are only interested in wavelet details of level 4 and 5, corresponding to our previous "wavelet scales" 4 and 5.

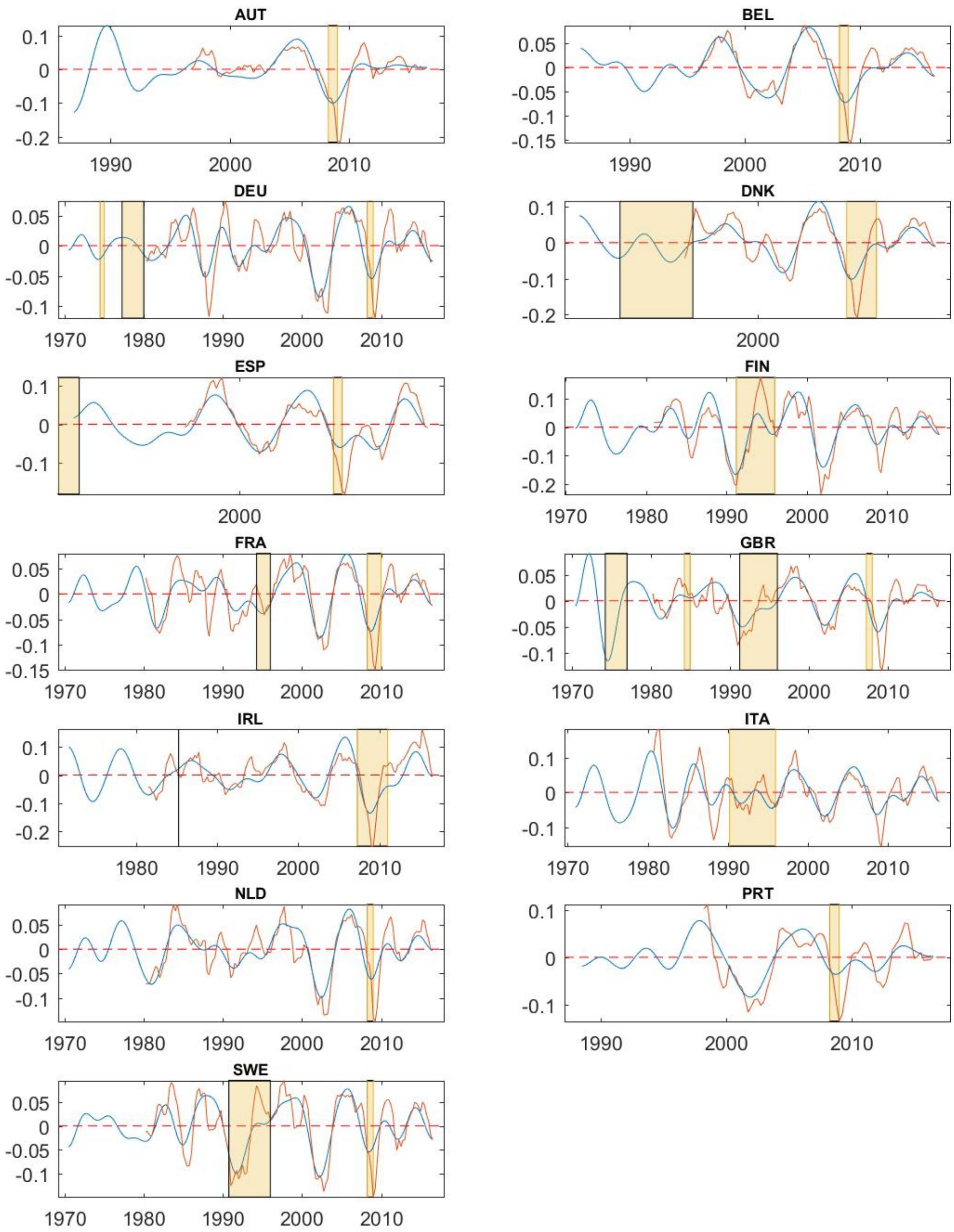

Figure 1: Wavelet-based financial cycle proxy $CI(\Omega_{45})$ plotted for each sample country. Blue curve exhibits cycle estimated with full sample. Red curve exhibits cycle estimated in expanding fashion, where we use 10 years worth of data in the first wavelet estimation. Yellow shaded areas represent Babecký et al. (2012) banking crises.

4 Out-of-sample early warning properties

In previous section we found out that wavelet scales 4 and 5 produced an optimal wavelet-based financial cycle proxy in terms of full sample early warning properties. In this section we use $CI(\Omega_{45})$ in two out-of-sample early warning exercises. First, we we perform a quasi real-time early warning exercise similar to one presented in Alessi and Detken (2011). The goal is to see how well our cycle proxy $CI(\Omega_{45})$ would have performed in detecting the financial crisis of 2007-2008 had it been used back then in real time.[15] Second, we perform a cross-country validation exercise in spirit of Lo Duca and Peltonen (2013) where countries are consecutively left out from the full sample early warning regression, after which implied crisis probability series and early warning statistics are calculated for the excluded country.

For comparison purposes we calculate results from the two out-of-sample exercises for two additional cycle proxies. First additional cycle proxy was introduced by Schüler et al. (2015), which we will here refer to as the *Composite Financial Cycle*, or CFC for short. The CFC is essentially a smoothed composite stress index constructed by using the same three macro-financial series as in this text, plus the government 10-year bond yield. The smoothing is done by using a band-pass filter in conjunction with spectral techniques. We choose to compare our cycle proxy to the CFC due to its close resemblance to our own proxy. The CFC used in this text is obtained through Bank of Finland's independent implementation of the model of Schüler et al. (2015).[16] Second additional cycle candidate is the credit-to-GDP gap, which we choose due its prominent role in Basel III regulation. Figures 2 and 3 in Appendix plot the additional cycle proxies.

4.1 Quasi real-time early warning exercise

In order to run the quasi real-time early warning exercise we divide our sample interval into two parts: a *pre-sample* ranging from 1970Q1- 1999Q4, and an *expanding sample* ranging from 2000Q1-2016Q2. This choice guarantees that we have at least 10 years worth of observations for each country to estimate the cycles for the first time. In addition, it divides the crisis periods evenly to both parts, leaving the periods of recent financial crisis into the expanding sample part. The detailed procedure is described in Algorithm 1.

We report results from this exercise using all three vulnerability horizons as well as five different values for policymaker's preference parameter θ around equal preference case $\theta = 0.50$ in Table 3. The wavelet-based financial cycle proxy $CI(\Omega_{45})$ has the best performance in vulnerability horizons of 4-12 and 8-16 quarters as it achieves the overall highest relative usefulness across the three cycle proxies. In the 4-12 quarter vulnerability horizon all three measures achieve their best performance at $\theta = 0.50$, with $CI(\Omega_{45})$ outperforming the other two clearly: for the

[15]In this work we don not consider publication lags.

[16]Details of this implementation are explained in Voutilainen (2017), which can be obtained from the author upon request. In summary, results there are generally in line with the ones obtained by Schüler et al. (2015). In particular, financial cycles are found to be longer than corresponding business cycles with most important co-movement taking place above period lengths of 8 years. Further, there are important heterogeneities between countries at higher cycle frequencies. However, the frequency bands identified in Voutilainen (2017) emphasize lower frequencies, resulting in slightly smoother cycles compared to ones identified by Schüler et al. (2015).

$CI(\Omega_{45})$ we have $U_r = 0.42$, whereas for the CFC $U_r = 0.15$ and for the credit-to-GDP gap $U_r = 0.09$. In the 8-16 quarter horizon results converge with all three measures showing decent performance. Still, the wavelet-based cycle continues to outperform: at it's best $CI(\Omega_{45})$ reaches $U_r = 0.59$ whereas credit-to-GDP gap obtains $U_r = 0.52$, both at $\theta = 0.50$. The CFC obtains its highest relative usefulness of 0.31 with a slight preference on predicting crises correctly more often with the cost of receiving more false flags ($\theta = 0.55$). With its highest U_r, $CI(\Omega_{45})$ exhibits very low false positive rate of 0.2 whereas the true positive rate is 0.8. On the other hand, both the CFC and credit-to-GDP gap exhibit true positive rates of 1.0, although with higher false positive rates than $CI(\Omega_{45})$ (0.7 and 0.5, respectively). However, if we wished to increase the true positive rate for $CI(\Omega_{45})$ we could increase the value of θ slightly to 0.55, in which case we would obtain a TRP/FPR combination of $1.0/0.6$ that still produces a fairly good relative usefulness of 0.38.

In the most distant vulnerability horizon of 12-20 quarters $CI(\Omega_{45})$ has decent success at higher preference for more true positives ($\theta = 0.60$) and outperforms the CFC clearly. However, here credit-to-GDP gap has the best performance with $U_r = 0.55$ at $\theta = 0.60$. Contrasting this back to the full sample early warning regressions in Table 2, the relatively poorer performance of our wavelet-based financial cycle proxy is not too big a surprise as it's performance seems to be best at closer vulnerability horizons of 4-12 and 8-16 quarters.

Overall, we deduce that $CI(\Omega_{45})$ has the best early warning performance across the thee cycle proxies as it outperforms in two out of three vulnerability horizons. Further, the highest overall relative usefulness $U_r = 0.59$ is obtained with $CI(\Omega_{45})$.

Algorithm 1: Expanding sample exercise

1 Set $T = 1999Q4$, so that $T + 1$ refers to period $2000Q1$, $T + 2$ to $2000Q2$, and so forth. Specify θ and the vulnerability horizon.

2 Estimate $CI(\Omega_{45})$, the CFC, and the credit-to-GDP gap series for each country using data up to T.

3 Fit logistic regression (5) using each of the three cycle proxies separately as the explanatory variable (including intercept). Notice that last date in the regression sample will be $T - h$, where h equals to either $12, 16,$ or 20, depending on the vulnerability horizon.

4 For each of the three cycle proxies, calculate the loss function for different values of τ and select the one minimizing loss function as optimal.

5 Using the obtained estimates and optimal τ, derive implied crisis probabilities as well as confusion matrix observations for period $T + 1$. Save the confusion matrix observations.

6 Set $T = T + 1$ and repeat from step 1.

7 Once each period in the expanding sample has been accounted for, calculate early warning signalling statistics using the saved confusion matrix observations.

8 Repeat process for different values of θ.

	$CI(\Omega_{45})$									CFC									Credit-to-GDP gap								
θ	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r
Vulnerability horizon 4-12 quarters																											
0.40	0	406	30	99	0.0	0.1	0.44	-0.04	-0.10	15	427	9	84	0.2	0.0	0.35	0.05	0.12	41	199	237	58	0.4	0.5	0.56	-0.16	-0.40
0.45	10	402	34	89	0.1	0.1	0.45	0.00	0.01	17	425	11	82	0.2	0.0	0.39	0.06	0.14	48	172	264	51	0.5	0.6	0.56	-0.11	-0.26
0.50	98	189	247	1	1.0	0.6	0.29	0.21	0.42	18	421	15	81	0.2	0.0	0.43	0.07	0.15	73	152	284	26	0.7	0.7	0.46	0.04	0.09
0.55	99	95	341	0	1.0	0.8	0.35	0.10	0.22	89	10	426	10	0.9	1.0	0.50	-0.05	-0.10	76	47	389	23	0.8	0.9	0.53	-0.08	-0.18
0.60	99	28	408	0	1.0	0.9	0.37	0.03	0.06	99	2	434	0	1.0	1.0	0.40	0.00	0.00	98	16	420	1	1.0	1.0	0.39	0.01	0.02
Vulnerability horizon 8-16 quarters																											
0.40	4	325	15	95	0.0	0.0	0.41	-0.01	-0.03	7	340	0	92	0.1	0.0	0.37	0.03	0.07	0	336	4	99	0.0	0.0	0.41	-0.01	-0.02
0.45	24	293	47	75	0.2	0.1	0.42	0.03	0.07	7	336	4	92	0.1	0.0	0.42	0.03	0.06	0	336	4	99	0.0	0.0	0.46	-0.01	-0.01
0.50	75	282	58	24	0.8	0.2	0.21	0.29	0.59	7	336	4	92	0.1	0.0	0.47	0.03	0.06	99	176	164	0	1.0	0.5	0.24	0.26	0.52
0.55	97	138	202	2	1.0	0.6	0.28	0.17	0.38	99	106	234	0	1.0	0.7	0.31	0.14	0.31	99	85	255	0	1.0	0.8	0.34	0.11	0.25
0.60	98	89	251	1	1.0	0.7	0.30	0.10	0.25	99	89	251	0	1.0	0.7	0.30	0.10	0.26	99	85	255	0	1.0	0.8	0.30	0.10	0.25
Vulnerability horizon 12-20 quarters																											
0.40	0	233	19	99	0.0	0.1	0.45	-0.05	-0.11	4	251	1	95	0.0	0.0	0.39	0.01	0.03	0	206	4	108	0.0	0.0	0.41	-0.01	-0.03
0.45	12	230	22	87	0.1	0.1	0.44	0.01	0.01	4	251	1	95	0.0	0.0	0.43	0.02	0.04	0	196	14	108	0.0	0.1	0.49	-0.04	-0.08
0.50	32	197	55	67	0.3	0.2	0.45	0.05	0.10	9	219	33	90	0.1	0.1	0.52	-0.02	-0.04	33	167	43	75	0.3	0.2	0.45	0.05	0.10
0.55	69	152	100	30	0.7	0.4	0.35	0.10	0.23	31	184	68	68	0.3	0.3	0.50	-0.05	-0.11	80	163	47	28	0.7	0.2	0.24	0.21	0.46
0.60	90	105	147	9	0.9	0.6	0.29	0.11	0.28	95	48	204	4	1.0	0.8	0.35	0.05	0.13	92	162	48	16	0.9	0.2	0.18	0.22	0.55

Table 3: *Confusion matrix, loss function, and usefulness values from the expanding sample exercise for each of the three cycle proxies and for different values of policymaker's preference parameter θ. Explained variable is Babecký et al. (2012) crises with different vulnerability horizons. Top block refers to vulnerability horizon of 1-3 years, middle block 2-4 years, and bottom block 3-5 years.*

4.2 Cross-country validation exercise

In this subsection we perform a cross-country validation exercise in the spirit of Lo Duca and Peltonen (2013) where countries are consecutively left out from the full sample early warning regression. That is, compared to the process in previous section here model parameters β_0, β_1, and τ are estimated once for a set of countries, and then used to construct model implied crisis probabilities for the country excluded from the estimation. The procedure is formally described in Algorithm 2.[17] In Table 4 we report the results for each country in a setting where we use the baseline vulnerability horizon of 8-16 quarters and have set the policymaker's preference parameter to 0.50, corresponding to equal preference case. Early warning statistics are reported for each excluded country separately, and the last two rows calculate mean and median over countries.[18] Table 8 in Appendix summarizes results by providing the mean and median values over different vulnerability horizons and values of θ.

From Table 4 we can deduce that at baseline vulnerability horizon and equal preference case our wavelet-based financial cycle proxy again outperforms the two contestants. The mean and median relative usefulness over all countries are 0.32 and 0.35, respectively, whereas for the CFC the same figures read 0.03 and 0.07, and for the credit-to-GDP gap 0.08 and 0.00. In addition, out of 13 countries the highest U_r is obtained by $CI(\Omega_{45})$ in 7 cases, by credit-to-GDP gap in 4 cases, and by the CFC in 2 cases. Furthermore, results for $CI(\Omega_{45})$ are more stable compared to the CFC and credit-to-GDP gap in the sense that it features non-zero true positives and true negatives for every country, meaning that it can correctly point out both alarms and non-alarms. Instead, for the CFC we have three cases (AUT, DEU, and PRT) where we don't issue a single correct alarm for a crisis. The same is true with credit-to-GDP gap for AUT, BEL, ESP, and PRT. Further, for DEU, FIN, and ITA credit-to-GDP gap issues a warning during all periods in the sample which results in both TPR and FPR equalling to 1.

From Table 8 we see that the competition between wavelet-based financial cycle proxy and credit-to-GDP gap is closer with the 4-12 quarter vulnerability horizon. With $\theta = 0.50$ the mean and median relative usefulness for $CI(\Omega_{45})$ read 0.23 and 0.18, respectively, whereas for credit-to-GDP gap they read 0.22 and 0.18, respectively. Around the equal preference case $CI(\Omega_{45})$ performs better at $\theta = 0.55$, but credit-to-GDP gap wins at $\theta = 0.45$. The CFC does not showcase any particular success in the given vulnerability horizon at any value of θ. However, as it was with the baseline vulnerability horizon, the wavelet-based cycle proxy outperforms the credit-to-GDP gap in terms of highest U_r for individual countries (7 vs. 6). Furthermore, the confusion matrix observations derived for the wavelet-based cycle proxy at $\theta = 0.50$ are again more stable in the

[17]The cross-country validation exercise involves deriving all three cycle proxies in an expanding fashion for the excluded country. For the wavelet-based financial cycle this procedure was described in Section 3.2. The expanding sample CFC is derived as in Schüler et al. (2015), i.e. band-pass filtering is conducted recursively adding one new observation at the time. However, spectral estimations and normalization of variables are done using full sample. Credit-to-GDP gap derived on an expanding sample is the so-called "Basel gap". That is, HP-filtering is conducted recursively adding one new observation at the time. We use 10 years worth of data in the first expanding sample estimation for all three cycle proxies.

[18]Mean and median values are not calculated for confusion matrix values (TP, TN, FP, and FN) as they are directly depended on the sample size of each individual country.

sense that with $CI(\Omega_{45})$ there are no case where we would constantly alarm/not alarm for a crisis, whereas with CFC this happens 5 times (AUT, BEL, DEU, SWE, and GBR) and with credit-to-GDP gap three times (DEU, DNK, and NLD).[19] Results for the vulnerability horizon 12-20 are generally poor; mean and median U_r values remain close to zero across all three cycle proxies.

In summary, our wavelet-based financial cycle proxy outperforms the two other contestants in the cross-country validation exercise by producing best early warning results in both 4-12 and 8-16 quarter vulnerability horizons. In the more distant vulnerability horizon of 12-20 quarters all three cycle proxies fail to showcase consistent success.

Algorithm 2: Cross-country validation

1 Specify θ and the vulnerability horizon.

2 Exclude one country from the sample. For rest of the countries, derive wavelet-based financial cycle $CI(\Omega_{45})$ as well as the CFC and the credit-to-GDP gap for full sample.

3 Next, run regression (5) on full sample using each of the three cycle proxies separately as the explanatory variable (including intercept). Notice that the actual regression sample will be somewhat shorter than the full sample, depending on the higher end of the vulnerability horizon. Obtain regression estimates, and calculate the implied crisis probabilities as well as the optimal τ as the one minimizing loss function.

4 Derive wavelet-based financial cycles as well as the CFC and the credit-to-GDP gap on an expanding sample for the excluded country.

5 Using the obtained estimates and optimal τ from step 3, derive implied crises probabilities as well as confusion matrix and usefulness values for the excluded country for all three explanatory variables.

6 Repeat by consecutively excluding different countries.

[19]These results are not shown but can be provided upon request.

	CI									CFC									Credit-to-GDP gap								
Country	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r
AUT	8	33	5	1	0.9	0.1	0.12	0.38	0.76	0	38	0	9	0.0	0.0	0.50	0.00	0.00	0	127	15	9	0.0	0.1	0.55	-0.05	-0.11
BEL	7	38	5	2	0.8	0.1	0.17	0.33	0.66	9	11	32	0	1.0	0.7	0.37	0.13	0.26	0	98	4	9	0.0	0.0	0.52	-0.02	-0.04
DEU	9	73	25	0	1.0	0.3	0.13	0.37	0.74	0	30	68	9	0.0	0.7	0.85	-0.35	-0.69	17	0	98	0	1.0	1.0	0.50	0.00	0.00
DNK	9	28	6	0	1.0	0.2	0.09	0.41	0.82	9	20	14	0	1.0	0.4	0.21	0.29	0.59	9	43	17	9	0.5	0.3	0.39	0.11	0.22
ESP	1	36	8	8	0.1	0.2	0.54	-0.04	-0.07	9	7	37	0	1.0	0.8	0.42	0.08	0.16	0	70	4	9	0.0	0.1	0.53	-0.03	-0.05
FIN	2	58	27	7	0.2	0.3	0.55	-0.05	-0.10	9	5	77	0	1.0	0.9	0.47	0.03	0.06	9	0	86	0	1.0	1.0	0.50	0.00	0.00
FRA	9	45	25	9	0.5	0.4	0.43	0.07	0.14	11	32	38	7	0.6	0.5	0.47	0.03	0.07	9	57	15	9	0.5	0.2	0.35	0.15	0.29
IRL	3	49	16	14	0.2	0.2	0.53	-0.03	-0.07	11	2	63	6	0.6	1.0	0.66	-0.16	-0.32	17	11	54	1	0.9	0.8	0.44	0.06	0.11
ITA	4	68	14	5	0.4	0.2	0.36	0.14	0.27	6	33	49	3	0.7	0.6	0.47	0.03	0.07	9	39	83	0	1.0	0.7	0.34	0.16	0.32
NLD	5	93	11	4	0.6	0.1	0.28	0.22	0.45	9	9	95	0	1.0	0.9	0.46	0.04	0.09	9	0	141	0	1.0	1.0	0.50	0.00	0.00
PRT	4	29	3	5	0.4	0.1	0.32	0.18	0.35	0	30	2	9	0.0	0.1	0.53	-0.03	-0.06	0	103	39	9	0.0	0.3	0.64	-0.14	-0.27
SWE	15	32	28	3	0.8	0.5	0.32	0.18	0.37	2	59	1	16	0.1	0.0	0.45	0.05	0.09	7	88	9	11	0.4	0.1	0.35	0.15	0.30
GBR	1	30	6	26	0.0	0.2	0.56	-0.06	-0.13	1	36	0	26	0.0	0.0	0.48	0.02	0.04	8	40	2	19	0.3	0.0	0.38	0.12	0.25
Mean	-	-	-	-	0.5	0.2	0.34	0.16	0.32	-	-	-	-	0.5	0.5	0.49	0.01	0.03	-	-	-	-	0.5	0.4	0.46	0.04	0.08
Median	-	-	-	-	0.5	0.2	0.32	0.18	0.35	-	-	-	-	0.6	0.6	0.47	0.03	0.07	-	-	-	-	0.5	0.3	0.50	0.00	0.00

Table 4: *Results from the cross-country validation exercise for all sample countries, using Babecký et al. (2012) crises dummies as explained variable, baseline vulnerability horizon 8-16 quarters, and policymaker's preference parameter* $\theta = 0.50$. *Mean and median over countries are not calculated for the confusion matrix observations as they depend on the sample size for each country.*

5 Robustness checks using alternative crisis dataset

In this section we perform robustness checks of the results derived above by employing an alternative banking crisis dataset, namely the one compiled by Detken et al. (2014). This crisis datasets is based on our baseline banking crisis dataset but incorporates some modifications. First, it excludes crises periods that are not systemic in nature. Second, it includes "would be crises", i.e. periods where banking crises would have taken place had it not been for some external event preventing it. See Section A.3 in Appendix for more detailed information. In Figure 4 in Appendix we plot the cycle proxy $CI(\Omega_{45})$ with Detken et al. (2014) crisis periods.

Table 5 exhibits the full sample regression results using Detken et al. (2014) crisis periods in similar fashion to Table 2. From the wavelet-based cycle proxies constructed using individual wavelet scales, $CI(\Omega_4)$ and $CI(\Omega_5)$ are again the only ones that have a statistically significant coefficients in all three vulnerability horizons. Furthermore, the best performance in terms of AUROC and relative usefulness are obtained using the cycle $CI(\Omega_{45})$ with vulnerability horizon of 8-16 quarters, where $AUROC = 0.66$ and $U_r = 0.25$. The outcome reinforces our findings that we can identify a robust financial cycle proxy – operating at mid-term period lengths – that is associated with onsets of financial crises.

Turning to the out-of-sample exercises, Table 6 presents results for the quasi real-time exercise in similar fashion to Table 3. We find that while $CI(\Omega_{45})$ loses by very close margin to the credit-to-GDP gap in the 4-12 quarter vulnerability horizon, it clearly outperforms the other two cycle proxies in the 8-16 quarter and 12-20 quarter vulnerability horizons. In the 4-12 quarter vulnerability horizon credit-to-GDP gap achieves maximum relative usefulness of 0.16 at $\theta = 0.50$, while the maximum relative usefulness for $CI(\Omega_{45})$ is also 0.16 but with $\theta = 0.45$. However, based on slightly higher absolute usefulness as well as more suitable TPR/FPR ratio, the credit-to-GDP gap can be interpreted to have better success in the given vulnerability horizon, although the overall performance is fairly modest across all three cycle proxies. In contrast, in the other two vulnerability horizons $CI(\Omega_{45})$ has very robust results over different values of θ (only exception being at 12-20 quarter horizon with $\theta = 0, 40$, where $U_r = -0.07$). Highest relative usefulness values for $CI(\Omega_{45})$ across the two horizons equal to 0.41 and 0.47, respectively (both with $\theta = 0.50$). In contrast, results in these two vulnerability horizon for the CFC and credit-to-GDP gap are quite poor, implying that $CI(\Omega_{45})$ shows clear outperformance in the expanding sample exercise with Detken et al. (2014) crisis definition.

Analogously to the expanding sample exercise, results from the cross-country validation exercise are clearly in favour of the wavelet-based financial cycle proxy when using Detken et al. (2014) crisis periods. Table 7 present results for the baseline vulnerability horizon and equal preference case in similar fashion to Table 4. However, we have omitted countries AUT and BEL as they feature no crisis periods in the Detken et al. (2014) dataset. Based on the mean and median values of U_r we deduce that $CI(\Omega_{45})$ outperforms the other two contestants. Further, $CI(\Omega_{45})$ has the highest U_r for 6 out of 11 countries, while the same is true for the CFC in 1 case and for the credit-to-GDP gap in 3 cases. In case of Sweden $CI(\Omega_{45})$ and the CFC tie with $U_r = 0.40$. Table

9 provides summary statistics of the mean and median values over different vulnerability horizons and values of θ in similar fashion to Table 8. We see that in the 8-16 quarter horizon values of θ on either sides of the equal preference case do not produce considerable success, confirming that $CI(\Omega_{45})$ is the best performer in this horizon. While credit-to-GDP gap outperforms the other two contestants in the 4-12 quarter horizon, in the 12-20 quarter horizon $CI(\Omega_{45})$ is again the best performer.

All in all, robustness checks using crisis definitions of Detken et al. (2014) suggest that our wavelet-based financial cycle proxy outperforms the other two contestants in all scenarios except for one[20], hence reinforcing our earlier results of it being a good early warning indicator for banking crises.

	$CI(\Omega_1)$	$CI(\Omega_2)$	$CI(\Omega_3)$	$CI(\Omega_4)$	$CI(\Omega_5)$	$CI(\Omega_6)$	$CI(\Omega_7)$	$CI(\Omega_{45})$	$CI(\Omega_{56})$	$CI(\Omega_{46})$	$CI(\Omega_{456})$
Vulnerability horizon 4-12 quarters											
Intercept	**−2.1**	**−2.1**	**−2.2**	**−2.2**	**−2.3**	**−2.1**	**−2.0**	**−2.3**	**−2.2**	**−2.2**	**−2.3**
CI	0.9	1.4	**6.8**	**14.0**	**22.3**	**20.1**	-3.8	12.1	**8.9**	**8.5**	**8.8**
AUROC	0.51	0.52	0.57	0.45	0.60	0.53	0.49	0.52	0.58	0.46	0.52
U_r	0.01	0.03	0.12	0.02	0.19	0.12	0.07	0.06	0.13	0.03	0.06
Obs	1518	1518	1518	1518	1518	1518	1518	1518	1518	1518	1518
Vulnerability horizon 8-16 quarters											
Intercept	**−2.1**	**−2.1**	**−2.1**	**−2.4**	**−2.7**	**−2.1**	**−2.2**	**−2.7**	**−2.6**	**−2.4**	**−2.7**
CI	0.2	-1.3	−5.1	**26.3**	**49.8**	3.9	2.5	**24.9**	**31.6**	**22.2**	**21.8**
AUROC	0.48	0.51	0.48	0.62	0.65	0.45	0.56	0.66	0.61	0.61	0.64
U_r	0.00	0.02	0.02	0.18	0.32	0.03	0.12	0.25	0.24	0.16	0.23
Obs	1422	1422	1422	1422	1422	1422	1422	1422	1422	1422	1422
Vulnerability horizon 12-20 quarters											
Intercept	**−2.1**	**−2.1**	**−2.1**	**−2.3**	**−2.7**	**−2.3**	**−2.5**	**−2.5**	**−2.7**	**−2.4**	**−2.3**
CI	0.4	0.7	1.9	**18.4**	**49.4**	**29.6**	8.8	**20.6**	**39.6**	**19.5**	**20.2**
AUROC	0.49	0.49	0.50	0.61	0.66	0.47	0.53	0.65	0.61	0.60	0.63
U_r	0.00	0.00	0.00	0.16	0.26	0.04	0.11	0.21	0.17	0.19	0.22
Obs	1330	1330	1330	1330	1330	1330	1330	1330	1330	1330	1330

Table 5: *Full sample early warning results using different weight selections for the wavelet-based financial cycle proxy CI. Explained variable is a dummy vector where values equal to 1 indicate Detken et al. (2014) banking crises at different vulnerability horizons. Top block refers to vulnerability horizon of 1-3 years, middle block 2-4 years, and bottom block 3-5 years. Usefulness measure U_r is derived using policymaker's preference parameter value $\theta = 0.50$. Wavelet estimations are done using full sample 1970Q1 - 2016Q2. For some countries series are shorter due to data availability issues. The last period for the pooled logistic regression will be either 2013Q2, 2012Q2, or 2011Q2, depending on the vulnerability horizon. Statistical significance of intercept and cycle coefficients at 5% and 1% levels are indicated by italic letters and bold letters, respectively.*

[20]The one being the 4-12 quarter vulnerability horizon case in cross-country validation.

	$CI(\Omega_{45})$									CFC									Credit-to-GDP gap								
θ	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r
Vulnerability horizon 4-12 quarters																											
0.40	0	399	0	77	0.0	0.0	0.40	0.00	0.00	2	399	0	75	0.0	0.0	0.39	0.01	0.03	51	167	232	26	0.7	0.6	0.48	-0.08	-0.21
0.45	16	384	15	61	0.2	0.0	0.38	0.07	0.16	2	396	3	75	0.0	0.0	0.44	0.01	0.02	58	156	243	19	0.8	0.6	0.45	0.00	0.01
0.50	45	215	184	32	0.6	0.5	0.44	0.06	0.12	5	331	68	72	0.1	0.2	0.55	-0.05	-0.11	61	147	252	16	0.8	0.6	0.42	0.08	0.16
0.55	77	12	387	0	1.0	1.0	0.44	0.01	0.03	77	30	369	0	1.0	0.9	0.42	0.03	0.08	63	114	285	14	0.8	0.7	0.42	0.03	0.06
0.60	77	0	399	0	1.0	1.0	0.40	0.00	0.00	77	13	386	0	1.0	1.0	0.39	0.01	0.03	74	48	351	3	1.0	0.9	0.38	0.02	0.06
Vulnerability horizon 8-16 quarters																											
0.40	22	322	25	47	0.3	0.1	0.32	0.08	0.21	7	346	1	62	0.1	0.0	0.36	0.04	0.10	0	341	6	69	0.0	0.0	0.41	-0.01	-0.03
0.45	25	310	37	44	0.4	0.1	0.35	0.10	0.23	8	346	1	61	0.1	0.0	0.40	0.05	0.11	0	339	8	69	0.0	0.0	0.46	-0.01	-0.03
0.50	46	258	89	23	0.7	0.3	0.29	0.21	0.41	9	345	2	60	0.1	0.0	0.44	0.06	0.12	9	157	190	60	0.1	0.5	0.71	-0.21	-0.42
0.55	67	121	226	2	1.0	0.7	0.31	0.14	0.31	69	43	304	0	1.0	0.9	0.39	0.06	0.12	50	20	327	19	0.7	0.9	0.58	-0.13	-0.28
0.60	69	72	275	0	1.0	0.8	0.32	0.08	0.21	69	26	321	0	1.0	0.9	0.37	0.03	0.07	50	20	327	19	0.7	0.9	0.54	-0.14	-0.36
Vulnerability horizon 12-20 quarters																											
0.40	0	286	13	64	0.0	0.0	0.43	-0.03	-0.07	6	299	0	58	0.1	0.0	0.36	0.04	0.09	0	297	2	64	0.0	0.0	0.40	-0.00	-0.01
0.45	40	255	44	24	0.6	0.1	0.25	0.20	0.45	6	298	1	58	0.1	0.0	0.41	0.04	0.09	0	292	7	64	0.0	0.0	0.46	-0.01	-0.03
0.50	46	224	75	18	0.7	0.3	0.27	0.23	0.47	6	289	10	58	0.1	0.0	0.47	0.03	0.06	0	288	11	64	0.0	0.0	0.52	-0.02	-0.04
0.55	51	177	122	13	0.8	0.4	0.30	0.15	0.34	64	71	228	0	1.0	0.8	0.34	0.11	0.24	35	73	226	29	0.5	0.8	0.59	-0.14	-0.31
0.60	61	105	194	3	1.0	0.6	0.29	0.11	0.28	64	36	263	0	1.0	0.9	0.35	0.05	0.12	35	28	271	29	0.5	0.9	0.63	-0.23	-0.59

Table 6: *Confusion matrix, loss function, and usefulness values from the expanding sample exercise for each of the three cycle proxies and for different values of policymaker's preference parameter θ. Explained variable is Detken et al. (2014) crises with different vulnerability horizons. Top block refers to vulnerability horizon of 1-3 years, middle block 2-4 years, and bottom block 3-5 years.*

	$CI(\Omega_{45})$									CFC									Credit-to-GDP gap								
Country	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r	TP	TN	FP	FN	TPR	FPR	L	U_a	U_r
DEU	1	79	13	8	0.1	0.1	0.52	-0.02	-0.03	0	92	0	9	0.0	0.0	0.50	0.00	0.00	9	2	128	0	1.0	1.0	0.49	0.01	0.02
DNK	9	19	17	0	1.0	0.5	0.24	0.26	0.53	9	13	23	0	1.0	0.6	0.32	0.18	0.36	9	45	17	9	0.5	0.3	0.39	0.11	0.23
ESP	9	21	20	0	1.0	0.5	0.24	0.26	0.51	9	12	29	0	1.0	0.7	0.35	0.15	0.29	0	72	0	9	0.0	0.0	0.50	0.00	0.00
FIN	6	31	56	3	0.7	0.6	0.49	0.01	0.02	0	84	0	9	0.0	0.0	0.50	0.00	0.00	9	0	88	0	1.0	1.0	0.50	0.00	0.00
FRA	11	33	33	7	0.6	0.5	0.44	0.06	0.11	5	52	14	13	0.3	0.2	0.47	0.03	0.07	18	0	68	0	1.0	1.0	0.50	0.00	0.00
IRL	9	58	35	0	1.0	0.4	0.19	0.31	0.62	1	85	8	8	0.1	0.1	0.49	0.01	0.03	9	0	94	0	1.0	1.0	0.50	0.00	0.00
ITA	2	55	43	7	0.2	0.4	0.61	-0.11	-0.22	9	5	93	0	1.0	0.9	0.47	0.03	0.05	8	94	44	1	0.9	0.3	0.21	0.29	0.57
NLD	7	59	13	7	0.5	0.2	0.34	0.16	0.32	9	62	10	5	0.6	0.1	0.25	0.25	0.50	14	0	109	0	1.0	1.0	0.50	0.00	0.00
PRT	9	9	3	0	1.0	0.3	0.13	0.38	0.75	5	3	9	4	0.6	0.8	0.60	-0.10	-0.19	11	96	14	7	0.6	0.1	0.26	0.24	0.48
SWE	13	42	20	5	0.7	0.3	0.30	0.20	0.40	8	59	3	10	0.4	0.0	0.30	0.20	0.40	18	0	99	0	1.0	1.0	0.50	0.00	0.00
GBR	3	44	12	15	0.2	0.2	0.52	-0.02	-0.05	8	29	27	10	0.4	0.5	0.52	-0.02	-0.04	18	0	66	0	1.0	1.0	0.50	0.00	0.00
Mean	-	-	-	-	0.6	0.4	0.36	0.14	0.27	-	-	-	-	0.5	0.4	0.43	0.07	0.13	-	-	-	-	0.8	0.7	0.44	0.06	0.12
Median	-	-	-	-	0.7	0.4	0.34	0.16	0.32	-	-	-	-	0.4	0.2	0.47	0.03	0.05	-	-	-	-	1.0	1.0	0.50	0.00	0.00

Table 7: *Results from the cross-country validation exercise using Detken et al. (2014) crises dummies as explained variable, baseline vulnerability horizon of 8-16 quarters, and policymaker's preference parameter* $\theta = 0.50$*. Results are not reported for AUT and BEL as they don't feature any crisis periods in the Detken et al. (2014) dataset. Mean and median over countries are not calculated for the confusion matrix values as they depend on the sample size for each country.*

6 Conclusion

In this paper we decomposed three key macro-financial variables – private credit, house prices, and stock prices – using wavelet multiresolution analysis. The resulting frequency-scale based sub-series represent movements in the original series at different period lengths, or equivalently at different frequencies. Based on their early warning properties, scales found most important are located in the period length ranges of 4-8 and 8-16 years. Sub-series operating at these ranges were aggregated into a composite index representing a proxy for the financial cycle. The composite index was found to be associated with onsets of financial crises. Thus, we provide further evidence to related literature (e.g. Drehmann et al., 2012, Schüler et al., 2015, and Verona, 2016) that relevant financial cycles operate at medium-term period lengths longer than the 2-8 years usually credited to the business cycle.

Furthermore, we showed that the identified financial cycle proxy exhibits good early warning properties in different out-of-sample exercises. We compared our cycle proxy to two other candidates: an alternative financial cycle proxy introduced by Schüler et al. (2015) as well as credit-to-GDP gap. We found that the wavelet-based financial cycle produced most accurate and robust results in early detection of banking crises in both exercises.

All in all, this paper contributes to the financial cycle literature by showcasing a fresh approach based on wavelets for constructing a proxy for the financial cycle. Further, our evidence suggests that the wavelet-based financial cycle proxy makes an useful addition to the macroprudential toolkit of central banks as it succeeds well in early warning detection of banking crises.

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A Appendix

A.1 Figures and tables

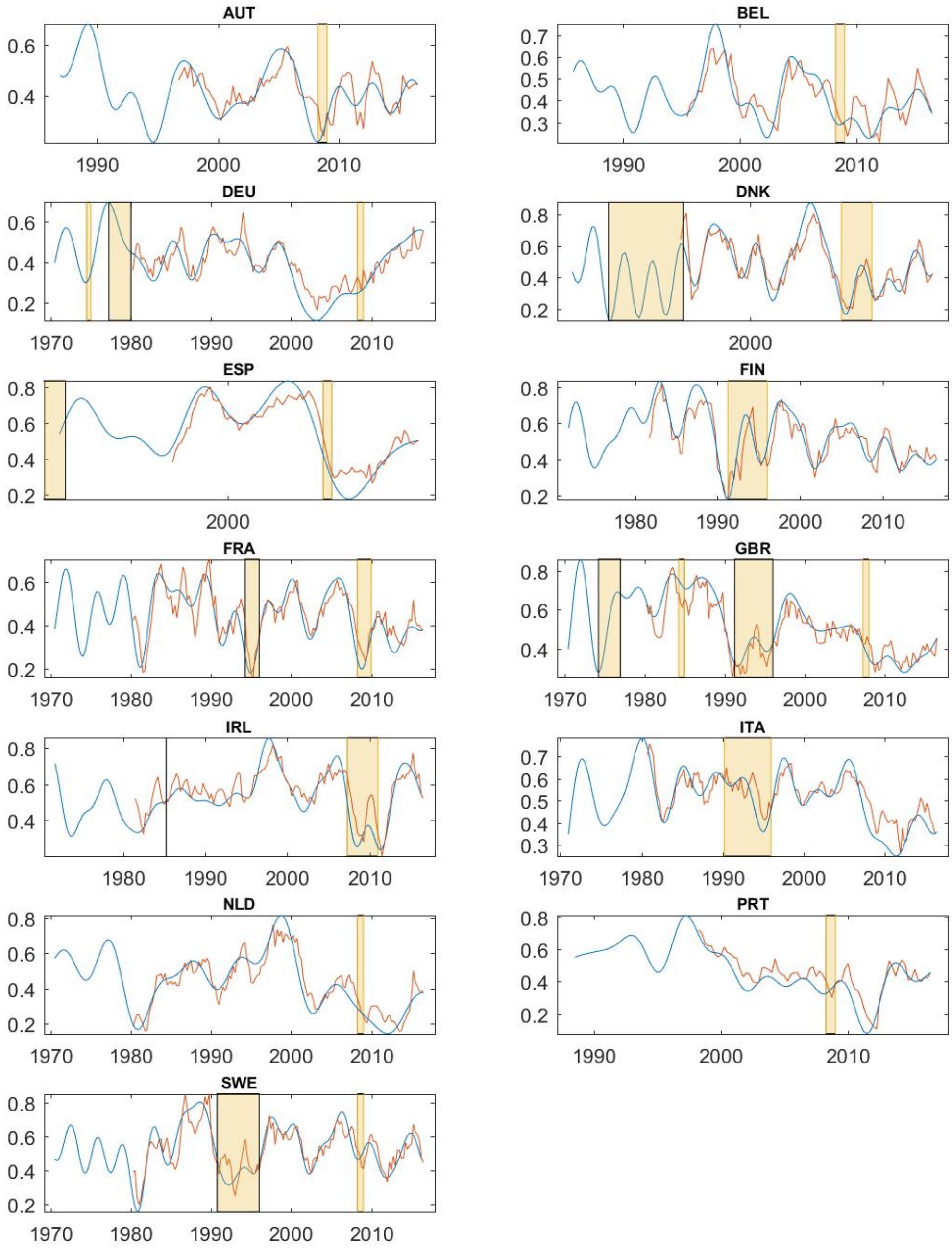

Figure 2: Financial cycle proxy Composite Financial Cycle (CFC) plotted for each sample country. CFC is constructed following the methodology of Schüler et al. (2015). Blue curve exhibits CFC calculated with full sample. Red curve exhibits CFC calculated in expanding fashion, where we use 10 years worth of data in the first band-pass filter estimation (see footnote 17). Yellow shaded areas represent Babecký et al. (2012) banking crises.

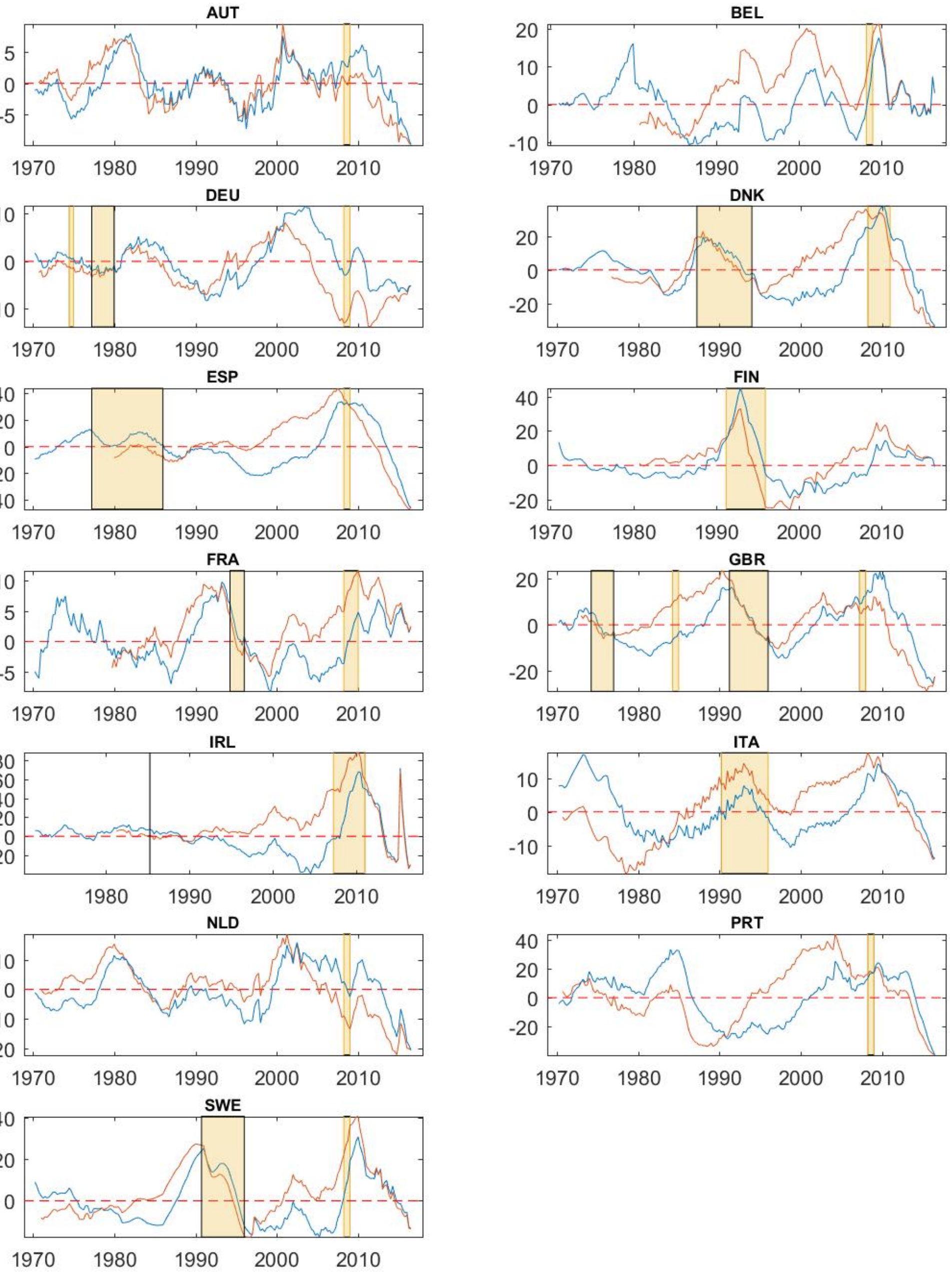

Figure 3: Credit-to-GDP gap (i.e. deviation of credit-to-GDP from its trend calculated using HP-filter with smoothing parameter of 400000) plotted for each sample country. Blue curve exhibits cycle calculated with full sample. Red curve exhibits cycle calculated in expanding fashion, where we use 10 years worth of data in the first HP-filter estimation (see footnote 17). Yellow shaded areas represent Babecký et al. (2012) banking crises.

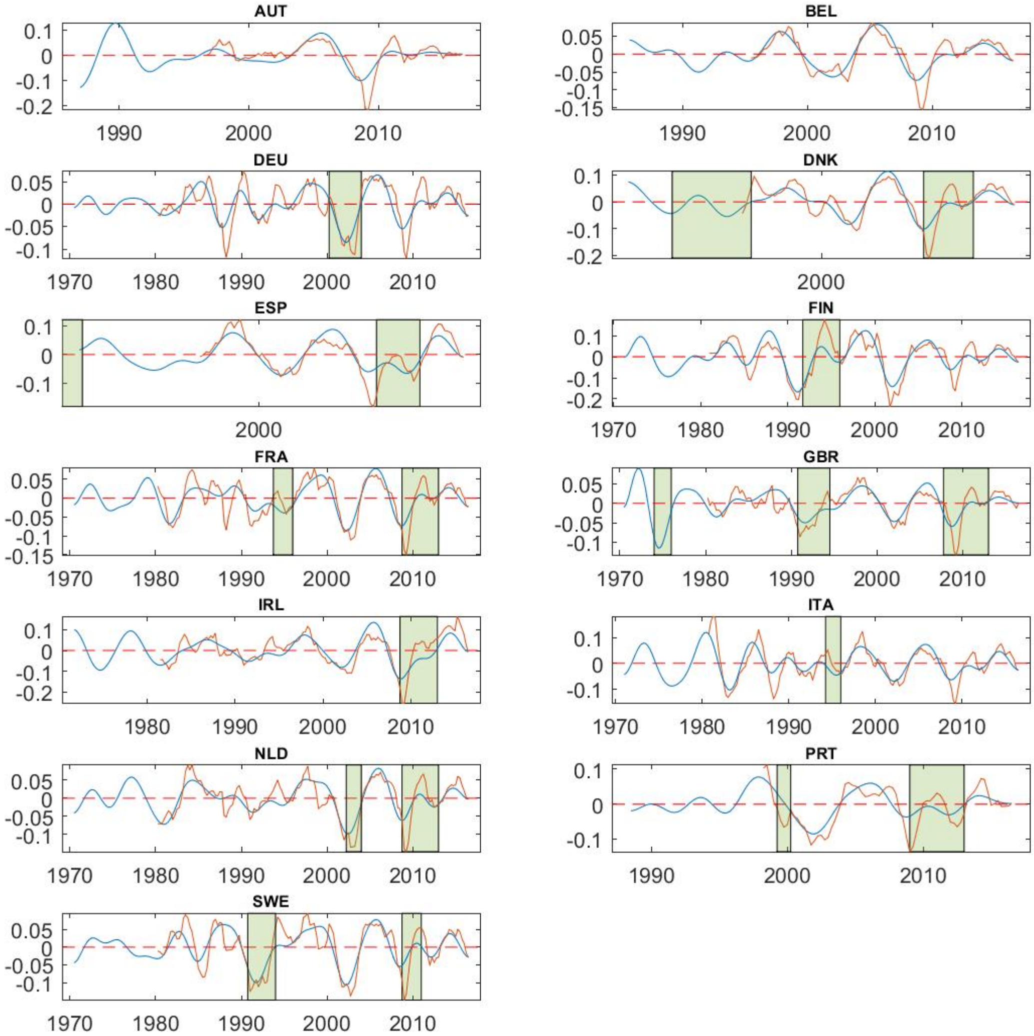

Figure 4: Wavelet-based financial cycle proxy $CI(\Omega_{45})$ plotted for each sample country. Blue curve exhibits cycle calculated with full sample and red curve exhibits cycle calculated in expanding fashion, where we use 10 years worth of data in the first wavelet estimation. Green shaded areas represent Detken et al. (2014) banking crises.

	$CI(\Omega_{45})$					CFC					Credit-to-GDP gap				
θ	TPR	FPR	L	U_a	U_r	TPR	FPR	L	U_a	U_r	TPR	FPR	L	U_a	U_r
Vulnerability horizon 4-12 quarters															
0.40	0.23 ∣ 0.17	0.15 ∣ 0.15	0.40 ∣ 0.41	0.00 ∣ -0.01	0.00 ∣ -0.03	0.13 ∣ 0.00	0.08 ∣ 0.00	0.40 ∣ 0.40	0.00 ∣ 0.00	0.01 ∣ 0.00	0.20 ∣ 0.00	0.11 ∣ 0.00	0.39 ∣ 0.40	0.01 ∣ 0.00	0.04 ∣ 0.00
0.45	0.38 ∣ 0.39	0.23 ∣ 0.20	0.40 ∣ 0.45	0.05 ∣ 0.00	0.10 ∣ 0.01	0.21 ∣ 0.00	0.14 ∣ 0.00	0.43 ∣ 0.45	0.02 ∣ 0.00	0.04 ∣ 0.00	0.35 ∣ 0.11	0.15 ∣ 0.16	0.37 ∣ 0.40	0.08 ∣ 0.05	0.17 ∣ 0.11
0.50	0.61 ∣ 0.67	0.38 ∣ 0.39	0.39 ∣ 0.41	0.11 ∣ 0.09	0.23 ∣ 0.18	0.46 ∣ 0.22	0.48 ∣ 0.52	0.51 ∣ 0.50	-0.01 ∣ 0.00	-0.03 ∣ 0.00	0.77 ∣ 1.00	0.56 ∣ 0.45	0.39 ∣ 0.41	0.11 ∣ 0.09	0.22 ∣ 0.18
0.55	0.73 ∣ 0.78	0.45 ∣ 0.48	0.35 ∣ 0.35	0.10 ∣ 0.10	0.21 ∣ 0.21	0.83 ∣ 1.00	0.82 ∣ 1.00	0.47 ∣ 0.45	-0.02 ∣ 0.00	-0.03 ∣ 0.00	0.91 ∣ 1.00	0.82 ∣ 1.00	0.42 ∣ 0.45	0.03 ∣ 0.00	0.07 ∣ 0.00
0.60	0.80 ∣ 1.00	0.70 ∣ 0.76	0.40 ∣ 0.33	0.00 ∣ 0.07	0.00 ∣ 0.18	0.98 ∣ 1.00	0.96 ∣ 1.00	0.40 ∣ 0.40	0.00 ∣ 0.00	0.01 ∣ 0.00	1.00 ∣ 1.00	0.99 ∣ 1.00	0.40 ∣ 0.40	0.00 ∣ 0.00	0.01 ∣ 0.00
Vulnerability horizon 8-16 quarters															
0.40	0.46 ∣ 0.44	0.15 ∣ 0.13	0.30 ∣ 0.32	0.10 ∣ 0.08	0.24 ∣ 0.21	0.07 ∣ 0.00	0.08 ∣ 0.00	0.42 ∣ 0.40	-0.02 ∣ 0.00	-0.05 ∣ 0.00	0.04 ∣ 0.00	0.03 ∣ 0.00	0.40 ∣ 0.40	-0.00 ∣ 0.00	-0.01 ∣ 0.00
0.45	0.50 ∣ 0.44	0.16 ∣ 0.17	0.31 ∣ 0.30	0.14 ∣ 0.15	0.30 ∣ 0.33	0.21 ∣ 0.00	0.14 ∣ 0.00	0.43 ∣ 0.45	0.02 ∣ 0.00	0.04 ∣ 0.00	0.11 ∣ 0.00	0.07 ∣ 0.04	0.44 ∣ 0.45	0.01 ∣ 0.00	0.02 ∣ 0.00
0.50	0.54 ∣ 0.50	0.21 ∣ 0.18	0.34 ∣ 0.32	0.16 ∣ 0.18	0.32 ∣ 0.35	0.54 ∣ 0.65	0.52 ∣ 0.60	0.49 ∣ 0.47	0.01 ∣ 0.03	0.03 ∣ 0.07	0.51 ∣ 0.50	0.43 ∣ 0.27	0.46 ∣ 0.50	0.04 ∣ 0.00	0.08 ∣ 0.00
0.55	0.76 ∣ 0.83	0.36 ∣ 0.36	0.30 ∣ 0.22	0.15 ∣ 0.23	0.34 ∣ 0.51	0.83 ∣ 1.00	0.84 ∣ 0.94	0.47 ∣ 0.45	-0.02 ∣ 0.00	-0.05 ∣ 0.00	0.90 ∣ 1.00	0.97 ∣ 1.00	0.49 ∣ 0.45	-0.04 ∣ 0.00	-0.09 ∣ 0.00
0.60	0.82 ∣ 1.00	0.64 ∣ 0.64	0.36 ∣ 0.37	0.04 ∣ 0.03	0.09 ∣ 0.08	0.92 ∣ 1.00	0.94 ∣ 0.97	0.42 ∣ 0.40	-0.02 ∣ 0.00	-0.05 ∣ 0.00	0.92 ∣ 1.00	0.98 ∣ 1.00	0.44 ∣ 0.40	-0.04 ∣ 0.00	-0.10 ∣ 0.00
Vulnerability horizon 12-20 quarters															
0.40	0.18 ∣ 0.11	0.14 ∣ 0.15	0.41 ∣ 0.40	-0.01 ∣ 0.00	-0.03 ∣ 0.00	0.04 ∣ 0.00	0.02 ∣ 0.00	0.39 ∣ 0.40	0.01 ∣ 0.00	0.01 ∣ 0.00	0.00 ∣ 0.00	0.02 ∣ 0.00	0.41 ∣ 0.40	-0.01 ∣ 0.00	-0.02 ∣ 0.00
0.45	0.26 ∣ 0.33	0.17 ∣ 0.16	0.43 ∣ 0.38	0.02 ∣ 0.07	0.05 ∣ 0.16	0.12 ∣ 0.00	0.08 ∣ 0.00	0.44 ∣ 0.45	0.01 ∣ 0.00	0.02 ∣ 0.00	0.03 ∣ 0.00	0.06 ∣ 0.00	0.47 ∣ 0.45	-0.02 ∣ 0.00	-0.04 ∣ 0.00
0.50	0.75 ∣ 0.78	0.74 ∣ 0.79	0.50 ∣ 0.50	0.00 ∣ 0.00	0.00 ∣ 0.00	0.65 ∣ 1.00	0.60 ∣ 0.78	0.47 ∣ 0.49	0.03 ∣ 0.01	0.06 ∣ 0.03	0.58 ∣ 0.69	0.60 ∣ 0.57	0.51 ∣ 0.50	-0.01 ∣ 0.00	-0.02 ∣ 0.00
0.55	0.77 ∣ 0.78	0.78 ∣ 0.80	0.48 ∣ 0.46	-0.03 ∣ -0.01	-0.06 ∣ -0.02	0.86 ∣ 1.00	0.80 ∣ 0.89	0.44 ∣ 0.42	0.01 ∣ 0.03	0.03 ∣ 0.06	0.90 ∣ 1.00	0.90 ∣ 1.00	0.46 ∣ 0.45	-0.01 ∣ 0.00	-0.02 ∣ 0.00
0.60	0.84 ∣ 0.89	0.82 ∣ 0.83	0.42 ∣ 0.42	-0.02 ∣ -0.02	-0.06 ∣ -0.05	0.92 ∣ 1.00	0.91 ∣ 0.94	0.41 ∣ 0.39	-0.01 ∣ 0.01	-0.03 ∣ 0.03	0.92 ∣ 1.00	0.98 ∣ 1.00	0.44 ∣ 0.40	-0.04 ∣ 0.00	-0.09 ∣ 0.00

Table 8: *Mean and median values over sample countries from the cross-country validation exercise for different values of the policymaker's preference parameter θ around the equal preference case, using Babecký et al. (2012) crises dummies as explained variable. Values are presented as mean ∣ median.*

	$CI(\Omega_{45})$					CFC					Credit-to-GDP gap				
θ	TPR	FPR	L	U_a	U_r	TPR	FPR	L	U_a	U_r	TPR	FPR	L	U_a	U_r
Vulnerability horizon 4-12 quarters															
0.40	0.01 ∣ 0.00	0.02 ∣ 0.00	0.41 ∣ 0.40	-0.01 ∣ 0.00	-0.02 ∣ 0.00	0.17 ∣ 0.00	0.09 ∣ 0.00	0.39 ∣ 0.40	0.01 ∣ 0.00	0.04 ∣ 0.00	0.25 ∣ 0.00	0.08 ∣ 0.00	0.35 ∣ 0.40	0.05 ∣ 0.00	0.13 ∣ 0.00
0.45	0.10 ∣ 0.00	0.10 ∣ 0.00	0.46 ∣ 0.45	-0.01 ∣ 0.00	-0.03 ∣ 0.00	0.21 ∣ 0.00	0.14 ∣ 0.00	0.43 ∣ 0.45	0.02 ∣ 0.00	0.04 ∣ 0.00	0.44 ∣ 0.11	0.18 ∣ 0.08	0.35 ∣ 0.40	0.10 ∣ 0.05	0.22 ∣ 0.11
0.50	0.59 ∣ 0.44	0.56 ∣ 0.54	0.49 ∣ 0.50	0.01 ∣ 0.00	0.03 ∣ 0.00	0.68 ∣ 0.80	0.59 ∣ 0.65	0.45 ∣ 0.50	0.05 ∣ 0.00	0.09 ∣ 0.00	0.90 ∣ 1.00	0.66 ∣ 0.55	0.38 ∣ 0.46	0.12 ∣ 0.04	0.25 ∣ 0.09
0.55	0.73 ∣ 1.00	0.73 ∣ 0.76	0.47 ∣ 0.45	-0.02 ∣ 0.00	-0.05 ∣ 0.00	0.81 ∣ 0.89	0.75 ∣ 0.88	0.45 ∣ 0.45	0.00 ∣ 0.00	0.01 ∣ 0.00	1.00 ∣ 1.00	0.87 ∣ 1.00	0.39 ∣ 0.45	0.06 ∣ 0.00	0.13 ∣ 0.00
0.60	0.86 ∣ 1.00	0.90 ∣ 1.00	0.45 ∣ 0.40	-0.05 ∣ 0.00	-0.12 ∣ 0.00	0.98 ∣ 1.00	0.95 ∣ 1.00	0.39 ∣ 0.40	0.01 ∣ 0.00	0.02 ∣ 0.00	1.00 ∣ 1.00	0.95 ∣ 1.00	0.38 ∣ 0.40	0.02 ∣ 0.00	0.05 ∣ 0.00
Vulnerability horizon 8-16 quarters															
0.40	0.30 ∣ 0.22	0.20 ∣ 0.18	0.40 ∣ 0.44	0.00 ∣ -0.04	0.00 ∣ -0.10	0.21 ∣ 0.00	0.11 ∣ 0.03	0.38 ∣ 0.40	0.02 ∣ 0.00	0.04 ∣ 0.00	0.11 ∣ 0.00	0.02 ∣ 0.00	0.37 ∣ 0.40	0.03 ∣ 0.00	0.08 ∣ 0.00
0.45	0.36 ∣ 0.22	0.26 ∣ 0.25	0.43 ∣ 0.48	0.02 ∣ -0.03	0.05 ∣ -0.06	0.25 ∣ 0.11	0.12 ∣ 0.05	0.41 ∣ 0.45	0.04 ∣ 0.00	0.10 ∣ 0.00	0.14 ∣ 0.00	0.02 ∣ 0.00	0.40 ∣ 0.45	0.05 ∣ 0.00	0.11 ∣ 0.00
0.50	0.64 ∣ 0.67	0.37 ∣ 0.38	0.36 ∣ 0.34	0.14 ∣ 0.16	0.27 ∣ 0.32	0.50 ∣ 0.44	0.36 ∣ 0.21	0.43 ∣ 0.47	0.07 ∣ 0.03	0.13 ∣ 0.05	0.82 ∣ 1.00	0.70 ∣ 1.00	0.44 ∣ 0.50	0.06 ∣ 0.00	0.12 ∣ 0.00
0.55	0.75 ∣ 0.78	0.47 ∣ 0.49	0.35 ∣ 0.35	0.10 ∣ 0.10	0.22 ∣ 0.21	0.88 ∣ 1.00	0.82 ∣ 0.94	0.43 ∣ 0.45	0.02 ∣ 0.00	0.04 ∣ 0.00	0.91 ∣ 1.00	0.96 ∣ 1.00	0.48 ∣ 0.45	-0.03 ∣ 0.00	-0.07 ∣ 0.00
0.60	0.80 ∣ 1.00	0.58 ∣ 0.62	0.35 ∣ 0.34	0.05 ∣ 0.06	0.11 ∣ 0.15	0.91 ∣ 1.00	0.85 ∣ 0.98	0.39 ∣ 0.40	0.01 ∣ 0.00	0.02 ∣ 0.00	0.91 ∣ 1.00	0.96 ∣ 1.00	0.44 ∣ 0.40	-0.04 ∣ 0.00	-0.09 ∣ 0.00
Vulnerability horizon 12-20 quarters															
0.40	0.21 ∣ 0.11	0.13 ∣ 0.13	0.39 ∣ 0.41	0.01 ∣ -0.01	0.02 ∣ -0.04	0.24 ∣ 0.00	0.11 ∣ 0.00	0.37 ∣ 0.40	0.03 ∣ 0.00	0.07 ∣ 0.00	0.00 ∣ 0.00	0.06 ∣ 0.00	0.43 ∣ 0.40	-0.03 ∣ 0.00	-0.09 ∣ 0.00
0.45	0.35 ∣ 0.28	0.18 ∣ 0.19	0.39 ∣ 0.41	0.06 ∣ 0.04	0.13 ∣ 0.09	0.39 ∣ 0.44	0.24 ∣ 0.20	0.40 ∣ 0.43	0.05 ∣ 0.02	0.10 ∣ 0.04	0.00 ∣ 0.00	0.09 ∣ 0.07	0.50 ∣ 0.49	-0.05 ∣ -0.04	-0.11 ∣ -0.08
0.50	0.63 ∣ 0.78	0.38 ∣ 0.41	0.37 ∣ 0.33	0.13 ∣ 0.17	0.26 ∣ 0.33	0.73 ∣ 0.89	0.58 ∣ 0.61	0.43 ∣ 0.45	0.07 ∣ 0.05	0.15 ∣ 0.11	0.08 ∣ 0.00	0.27 ∣ 0.10	0.60 ∣ 0.55	-0.10 ∣ -0.05	-0.19 ∣ -0.10
0.55	0.81 ∣ 0.83	0.52 ∣ 0.57	0.34 ∣ 0.32	0.11 ∣ 0.13	0.25 ∣ 0.28	0.93 ∣ 1.00	0.82 ∣ 0.92	0.41 ∣ 0.44	0.04 ∣ 0.01	0.09 ∣ 0.01	0.43 ∣ 0.10	0.71 ∣ 0.71	0.63 ∣ 0.65	-0.18 ∣ -0.20	-0.40 ∣ -0.44
0.60	0.95 ∣ 1.00	0.67 ∣ 0.73	0.30 ∣ 0.30	0.10 ∣ 0.10	0.24 ∣ 0.25	0.94 ∣ 1.00	0.85 ∣ 0.99	0.38 ∣ 0.40	0.02 ∣ 0.00	0.06 ∣ 0.00	0.66 ∣ 1.00	0.86 ∣ 1.00	0.55 ∣ 0.40	-0.15 ∣ 0.00	-0.37 ∣ 0.00

Table 9: *Mean and median values over sample countries from the cross-country validation exercise for different values of the policymaker's preference parameter θ around the equal preference case, using Detken et al. (2014) crises dummies as explained variable. AUT and BEL are excluded from the sample since they don't feature crisis periods in Detken et al. (2014) dataset. Values are presented as mean ∣ median.*

Country	Description	Original unit	Time-span	Original frequency	(Dis-) aggregation	Source
AUT	Credit to non-financial private sector, ex. trade credit, not seasonally adjusted	Euro, bil.	1970Q1-2016Q2	Quarterly	-	BIS Credit Statistics, DBSOnline
BEL	— ” —	— ” —	1970Q4-2016Q2	— ” —	-	— ” —
DEU	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
DNK	— ” —	Danish Krone, bil.	1970Q1-2016Q2	— ” —	-	— ” —
ESP	— ” —	Euro, bil.	1970Q1-2016Q2	— ” —	-	— ” —
FIN	— ” —	— ” —	1970Q4-2016Q2	— ” —	-	— ” —
FRA	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
GBR	— ” —	Pound sterling, bil.	1970Q1-2016Q2	— ” —	-	— ” —
IRL	— ” —	Euro, bil.	1970Q4-2016Q1	— ” —	-	— ” —
ITA	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
NLD	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
PRT	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
SWE	— ” —	Swedish Krona, bil.	1970Q1-2016Q2	— ” —	-	— ” —

Table 10: Information for private credit raw series.

Country	Description	Original unit	Time-span	Original frequency	(Dis-) aggregation	Source
AUT	Residential property prices, new and existing dwellings, whole country, neither SA nor WA	Index 2007=100	1986Q3-2016Q2	Quarterly	-	ECB SDW Residential Property Price Index Statistics
BEL	Residential property prices, long series, NSA	Index 1995=100	1970Q1-2016Q2	— ” —	-	BIS Residential Prices Statistics, DBSOnline
DEU	— ” —	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
DNK	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
ESP	— ” —	— ” —	1971Q1-2016Q2	— ” —	-	— ” —
FIN	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
FRA	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
GBR	— ” —	— ” —	1970Q1-2016Q2	— ” —	-	— ” —
IRL	— ” —	— ” —	1970Q4-2016Q1	— ” —	-	— ” —
ITA	— ” —	— ” —	1970Q3-2016Q2	— ” —	-	— ” —
NLD	— ” —	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
PRT	Residential property prices, new and existing dwellings, whole country, neither SA nor WA	Index 2007=100	1988Q1-2016Q2	— ” —	-	ECB SDW Residential Property Price Index Statistics
SWE	Residential property prices, long series, NSA	Index 2007=100	1970Q1-2016Q2	— ” —	-	BIS Residential Prices Statistics, DBSOnline

Table 11: Information for house price raw series.

Country	Description	Original unit	Time-span	Original frequency	(Dis-) aggregation	Source
AUT	VSE WBI index	Index	1970Q1-2016Q3	Quarterly	-	OECD Main Economic Indicators
BEL	All Shares index	— ” —	1985Q2-2016Q3	— ” —	-	— ” —
DEU	CDAX index	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
DNK	KAX CSE All Shares index	— ” —	1983Q1-2016Q3	— ” —	-	— ” —
ESP	IGBM general index	— ” —	1985Q1-2016Q3	— ” —	-	— ” —
FIN	HEX All Share index	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
FRA	Paris Stock Exchange SBF 250 ind	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
GBR	GBR FTSE 100 share price index	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
IRL	ISEQ Overall index	— ” —	1970Q4-2016Q3	— ” —	-	— ” —
ITA	ISE MIB Storico Generale	— ” —	1970Q3-2016Q3	— ” —	-	— ” —
NLD	AEX all shares	— ” —	1970Q1-2016Q3	— ” —	-	— ” —
PRT	BVL general index	— ” —	1988Q1-2016Q3	— ” —	-	— ” —
SWE	AFGX Index	— ” —	1970Q1-2016Q3	— ” —	-	— ” —

Table 12: Information for stock price raw series.

A.2 Average wavelet coherencies

In order to gain insight on the co-movement of our three series – private credit, house prices, and stock prices – at different period lengths, we turn to wavelet coherence analysis. Similar to coherence in spectral analysis, wavelet coherence determines correlation between two series at certain period length (or equivalently at different frequency). The advantage is that instead of pure frequency-domain resolution we also obtain resolution in time-domain, which can provide more accurate characterization of the nature of cyclical behaviour. Formally, wavelet coherence between series j and k is given by formula

$$WC_{jk}(a,b) = \frac{|S(C_j(a,b)C_k(a,b))|^2}{S(|C_j(a,b)|^2)S(|C_k(a,b)|^2))}$$

where $C_j(a,b)$ and $C_k(a,b)$ denote the continuous wavelet transform[21] of variables i and k, at scale a and at time position b. We use Matlab's Wavelet Toolbox to calculate wavelet coherencies between the three variables and take the average of the coherences at each frequency-time combination, i.e.

$$AWC(a,b) = \frac{1}{3}\sum_{j\neq k} WC_{jk}(a,b) \quad j,k = 1,2,3$$

$AWC(a,b)$ summarizes the co-movement of our three series at different period lengths over time. Figure 5 presents $AWC(a,b)$ for each country in the sample. Yellow colors indicate high average co-movement among the series, whereas blue colors indicate little co-movement. The results show that most of the common movement among the series is indeed located at higher scale ranges. Germany, Denmark, Finland, Italy, and Sweden feature consistent co-movement over time with period lengths of 8 years and above (corresponding to wavelet scales 5 and 6). Ireland and UK feature co-movement around same period lengths but only in parts of the sample. Most countries also feature some co-movement around periodicities of 4-8 years, but these patches are more localized. Seasonal cyclical behaviour is also visible for all countries. Years 2-4 (corresponding to wavelet scale 3) exhibits least co-movement. All in all, wavelet coherence analysis offers support for the view according to which wavelet scales 4-5 (4-8 and 8-16 years) are most relevant ones for our analysis. Scale 6 (16-32 years) might also be relevant to some extent, but it is to be noted that cycles much above 16 years can already become poorly estimated, which is seen in Figure 5 with the dashed white line indicating the cone of influence from end-point problems; cycles greater than 16 years fall mostly off the cone.

[21] Here we use *continuous wavelet transform* (CWT) instead the discrete wavelet transform used in the main text as wavelt coherency is readily obtained via CWT using Matlab's Wavelet toolbox.

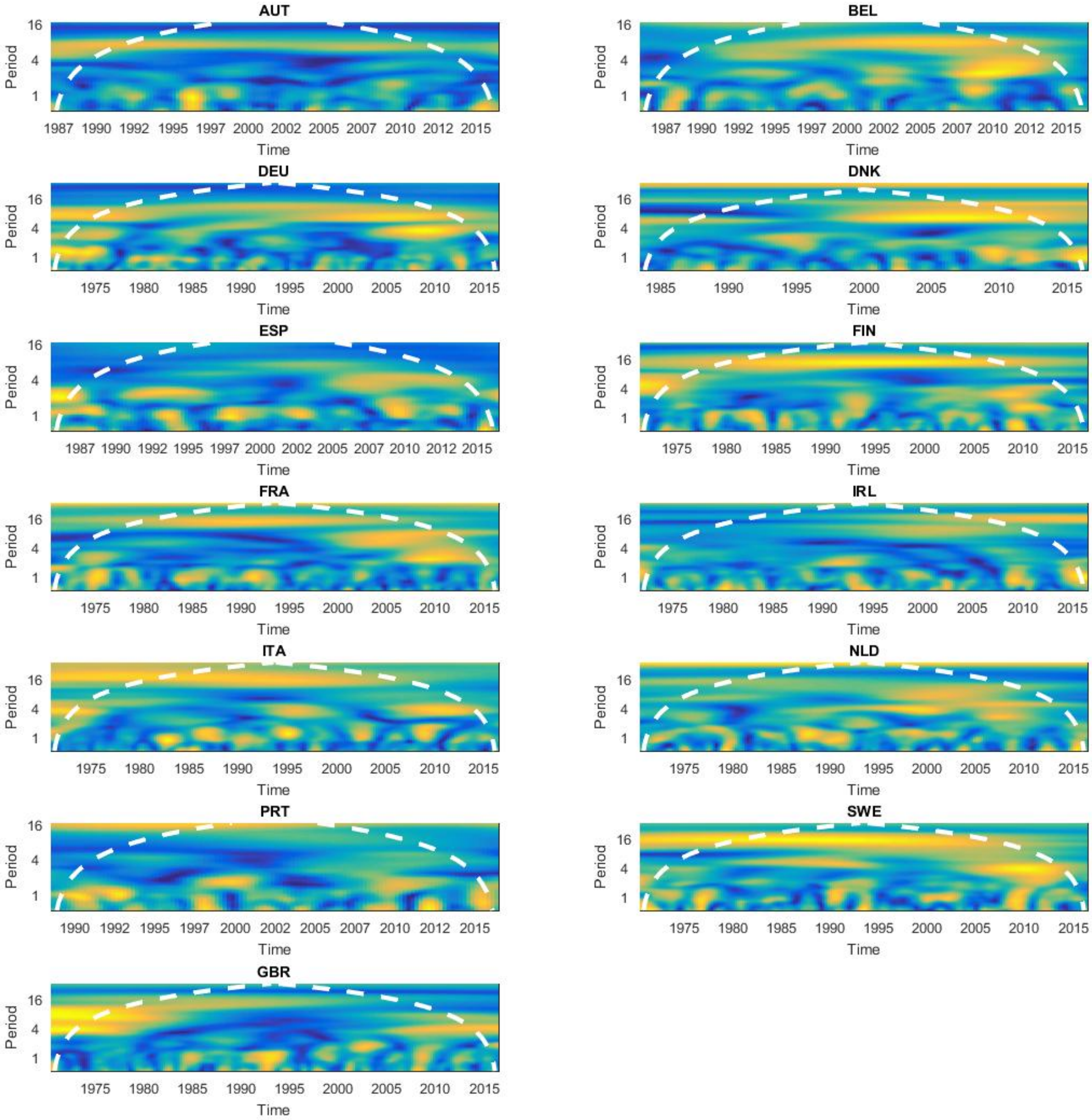

Figure 5: Averages of wavelet coherencies. Period lengths of common co-movement in the y-axis is measured in years. Yellow areas indicate high co-movement among variables whereas blue areas indicate low co-movement. Dashed white lines point the cone of influence from end-point problems. Values outside the cone might be poorly estimated.

A.3 Information about the crisis datasets

This section presents information on the banking crisis datasets compiled by Babecký et al. (2012) and Detken et al. (2014). Specifically, we provide definitions for periods to be indicated as having a banking crisis as well as the exact crisis periods in both datasets.[22]

- **Babecký et al. (2012)**: Significant signs of financial distress in the banking system as evidenced by bank runs or losses (non-performing loans above 20% or bank closures amounting to at least 20% of banking system assets).
- **Detken et al. (2014)**: Same datasets as Babecký et al. (2012) but amended with following changes. Non-systemic banking crises and crises not associated with credit cycle are excluded. "Would be crisis" added to periods where domestic developments related to credit cycle could have caused a systemic baking crisis had it not been for policy actions or an external event that dampened the financial cycle.

Country	Babecký et al. (2012)	Detken et al. (2014)
AUT	2008Q1-2008Q4	-
BEL	2008Q1-2008Q4	-
DEU	1974Q2-1974Q4	-
	1977Q1-1979Q4	-
	-	2000Q1-2003Q4
	2008Q1-2008Q4	-
DNK	1987Q1-1993Q4	1987Q1-1993Q4
	2008Q1-2010Q4	2008Q3-2012Q4
ESP	1977Q1-1985Q4	1978Q1-1985Q3
	2008Q1-2008Q4	2009Q2-2012Q4
FIN	1991Q1-1995Q4	1991Q3-1995Q4
FRA	1994Q1-1995Q4	1993Q3-1995Q4
	2008Q1-2009Q4	2008Q3-2012Q4
IRL	1985Q1-1985Q1	-
	2007Q1-2010Q4	2008Q3-2012Q4
ITA	1990Q1-1995Q4	1994Q1-1995Q4
NLD	-	2002Q1-2003Q4 (A)
	2008Q1-2008Q4	2008Q3-2012Q4
PRT	-	1999Q1-2000Q1 (A)
	2008Q1-2008Q4	2008Q4-2012Q4
SWE	1990Q3-1995Q4	1990Q3-1993Q4
	2008Q1-2008Q4	2008Q3-2010Q4
GBR	1974Q1-1976Q4	1973Q4-1975Q4
	1984Q1-1984Q4	-
	1991Q1-1995Q4	1990Q3-1994Q2
	2007Q1-2007Q4	2007Q3-2012Q4

Table 13: Crisis periods for the sample countries in Babecký et al. (2012) and Detken et al. (2014) crisis datasets. For the Detken et al. (2014) dataset (A) denotes domestic developments related to the credit cycle that could well have caused a systemic banking crisis had it not been for policy action or an external event that dampened the credit cycle.

[22] Definitions and the table are gratefully taken from Tölö et al. (2017).

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